Firm Exit and Financial Frictions*

Gideon Bornstein† Laura Castillo-Martinez‡

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Abstract

This paper studies the role of financial frictions in firm exit behavior and develops a model to assess the costs of inefficient firm exit. Using European firm-level data, we document a positive relationship between firm-level exit and leverage, controlling for non-financial firm characteristics. We find that this relationship is significantly stronger during recessions. We then construct a firm dynamics model with financial frictions. The model endogenously delivers a stronger relationship between firm exit and leverage during recessions. We show that the correlation between firm exit and a firm's financial condition prior to exit is informative of the degree of financial frictions in the economy. Using a calibrated version of the model, we assess the costs of inefficient firm exit due to financial frictions and study the effectiveness of different government interventions during recessions.

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†Wharton School. Email: gideonbo@wharton.upenn.edu.

‡Duke University and CEPR. Email: l.castillo-martinez@duke.edu.

1 Introduction

Firm exit is a significant contributor to overall job destruction. About 20% of job destruction in the US are associated with businesses that shut down.[1] In addition to jobs, there are also firm-specific forms of intangible capital that are inevitably lost when firms cease operation. This includes inputs that, while challenging to measure, have become a fast-growing share of corporate capital stock: brand value, customer base, and firm-specific human capital. Yet the underlying causes of firm exit remain a relatively unexplored question in macroeconomics.[2] This paper aims to fill the gap in the literature by studying the role of financial soundness in firm survival.

The canonical firm dynamics model puts productivity at the center of the firm's decision problem. In the absence of frictions, long-run analysis of technical efficiency suffices to determine firm turnover. At the same time, survey responses by exiting firm owners place financing hurdles among the top factors behind business failure.[3] Consistent with this view, policy efforts during recessions are often directed towards extending liquidity. The underlying goal is to prevent the exit of firms that, while struggling due to temporarily diminished cashflows, come across as viable.[4] In this line, novel data access on the characteristics of exiting firms allows us to study the financial roots of firm exit. As a preliminary exercise, Figure 1 plots average exit rates across firms grouped according to their financial health. For a given industry-year, firms that are more leveraged and hold fewer liquid assets are more likely to exit. While unsurprising, this pattern raises a set of important, related questions: is financial soundness a key driver of firm exit beyond fundamentals? Does this imply the presence of financial market imperfections? What share of firm exit is, thus, efficient?

This paper makes three contributions to our understanding of the interaction between financial frictions and firm exit, in general, and during recessions, more specifically. First, it exploits U.S. Census micro-data to characterize the sensitivity of firm exit to leverage as a function of the business cycle, while controlling for non-financial characteristics. Second, it establishes that the conditional correlation between firm propensity to exit and debt levels is informative of

[1] Authors' calculation using Business Dynamics Statistics (1978–2018). The share of job destruction due to establishment (rather than firm) exit is even higher, exceeding one third.

[2] Researchers have documented stark differences in exit rates across locations, industries, and firm age. There is also some albeit inconclusive evidence on the pattern of firm exit over the business cycle. However, little else is known about the specific factors that prompt firms' death.

[3] The US Small Business Administration, using data provided by the 2015 Annual Survey of Entrepreneurs and the 2007 Survey of Business Owners, points at low sales/cash flows and business credit as the main two non-personal factors explaining firm closures. The role of business credit, however, was significantly higher at the onset of the Great Recession: 5% of owners selected credit access as a reason to close in 2015 down from 14% in 2007. More recently, CB Insights collects 101 startup post-mortems, showing that lack of liquidity ranks second among the most popular reasons startups offer for their failure

[4] Such policies have been particularly popular during the Covid-19 crisis; especially in the form of tax relief, grants and cash transfers, and guaranteed loans which simplify access to credit, among others.

Figure 1: Average Exit Rates by Financial Health

Higher Leverage ↓ / Lower Liquidity →					
	2.5	1.7	1.8	1.5	2.4
	2.2	2.2	1.6	1.5	2.1
	2.6	2.1	2.1	1.8	2.9
	3.4	3.9	2.5	3.2	3.7
	5.9	5.3	5.5	6.4	7.2

Notes: This figure summarizes average exit rates when firms are grouped based on financial health using data for Ireland, Italy and Portugal for the 2002-2016 period provided by ORBIS. Buckets are calculated as follows: for every industry-year-country triplet, firms are first sorted into quintiles based on their leverage (measured as the debt to assets ratio). Firms in each of these quintiles are then sorted further into additional quintiles based on their liquidity ratio (measured as (current assets - inventories)/current liabilities). We measure the average exit rate of firms within each bucket. We then average bucket exit rates across industries, years and countries.

the presence of inefficient exit in a simple firm dynamics model with financial frictions. Third, it uses a quantitative version of the model to evaluate the cost of financial frictions strictly through firm exit in the U.S. economy. With the goal of alleviating these market imperfections, the paper then proposes and evaluates alternative policy instruments.

Section 2 aims to shed light empirically on the key drivers of firm exit. Using balance sheet and profit and loss statement data at the firm-level, we characterize the sensitivity of firms' propensity to exit to financial characteristics, and document how this sensitivity varies over the business cycle. The novel access to detailed financial data allows us to disentangle insolvency concerns, defined as the long-run sustainability of business operations, from liquidity pressures, defined as short-term restrictions in the access to credit markets. The hypothesis is that the latter play a sizable role, especially during periods of economic slowdown.

Preliminary findings, using European data by ORBIS, show that financial measures such as the debt-to-assets ratio, the quick ratio and the average cost of debt are strong predictors of firms' propensity to exit, contributing to a substantial increase in the explained variation. As expected, firms with more leverage, less liquidity and higher financing costs are more likely to exit the market. Having controlled for firm fundamentals that include age, industry, size, labor productivity and capital stock, we take this as suggestive evidence of the presence of financial frictions in firms' exit decisions.

During recessions, there is a greater share of financially constrained firms. In line with this view, the correlation between financial health and likelihood of survival strengthens in periods of economic downturn i.e. years during which the unemployment rate increases. This result is not only statistically significant but economically relevant. During the Great Financial Crisis, the difference in the predicted exit rate between firms with debt-to-assets ratio one standard deviation apart almost doubled relative to normal times.

Following the empirical analysis, section 3 develops a quantitative general equilibrium business cycle model to study the effect of financial distress on firm exit. The model has a continuum of heterogeneous firms that produce differentiated products. We allow for firm entry and exit. Each period, firms face stochastic fixed operating costs which they are required to pay in order to remain in the market. If profits are not enough to cover the cost, firms can raise funds from financial intermediaries that operate internationally. Firm owners will always choose to borrow, if available, and keep the firm in the market.

Financial markets, however, are incomplete and firms can default on their non-contingent debt. If a firm defaults, financial intermediaries take over the firm, which can then be run by new owners but at a fraction of its initial productivity. As a result, firms face an endogenous borrowing limit that depends not only on the probability of default but also on the value of the firm under new ownership.

Our environment is unique in that firm default does not strictly imply firm exit. In fact, if the new owner is able to run the firm at full productivity, firm exit is independent of default and determined only by the present discounted sum of net profits. That is, the equilibrium outcome coincides with that of the Hopenhayn (1992) complete markets economy. As productivity under new ownership decreases, the relationship between default and exit tightens for a given productivity level. It is less likely that new owners find it profitable to run firms following default. Thus, firms with higher debt burdens are more prone to exit as they default more often, other things equal. The conditional covariance of a firm's propensity to exit with its debt holdings provides a measure of the degree of market imperfections in the model.

Equipped with a rich set of moments from the micro-data, section 4 quantifies the cost of financial frictions due to firm exit. In the current version, we calibrate our model using a combination of U.S. Census Bureau data and the European ORBIS data used in section 2. Our preliminary results show that losses are sizable. We estimate that, out of an overall exit rate of 6.6%, 4.9 percentage points correspond to exit driven by financial frictions and eliminating these would boost annual consumption by 2.1%. In future work, we show how the cost of financial frictions depends on aggregate conditions and on the magnitude of entry costs.

Finally, section 5 considers the role of government interventions. We study the effectiveness of different policies aimed at alleviating financial market imperfections. The model generates a trade-off between the prevention of firm exit of long term viable firms at the expense of an increased risk of keeping unproductive firms alive. This implies the government cannot fully restore the first best even with unlimited resources. Thus, the optimal design of firm subsidies becomes an interesting question when resources are finite.

Related Literature This paper contributes to three strands of literature. First, it contributes to the empirical literature on firm dynamics. Second, it contributes to the theoretical literature on firm dynamics with financial frictions. Third, it contributes to the literature on cash-flow based borrowing constraints.

A narrow literature has investigated empirically the characteristics of exiting firms. Lee and Mukoyama (2015) show, using the U.S. Census Annual Survey of Manufactures, that plant entry and exit vary over the business cycle, with substantially larger differences in the case of entry. Foster et al. (2016) exploit the same micro-data to document that the intensity of productivity-enhancing resource reallocation that characterizes recessions fell during the Great Recession. Leibovici and Wiczer (2023) exploit delinquency rates collected by Dun and Bradstreet to argue that the increase in exit rates during such period is mainly driven by firms under financial stress.

In an international context, Castillo-Martinez (2020) provides evidence that selection among exiting firms during recessions depends on a country's exchange rate regime. More recently, Gourinchas et al. (2022) develop a framework to predict individual firm exits. In the context of the Covid19 crisis, they document that cross-sector and cross-country heterogeneity is explained by exposure to sector-specific shocks and firm financial health. The main contribution relative to this empirical literature is that we are able to observe and study the financial characteristics of exiting firms, and how these vary over the business cycle.

In terms of modeling choices, the paper is closest to the work of Cooley and Quadrini (2001), who introduce financial frictions to the firm dynamics model of Hopenhayn (1992). The friction they study arises from limited commitment in debt contracts. The paper is also related to Albuquerque and Hopenhayn (2004), who study how enforcement problems affect firm dynamics when firms can trade using a full set of state-contingent assets. Relative to these papers, this paper introduces aggregate fluctuations into the theoretical framework and studies how financial frictions shape exit patterns over the business cycle.

Two related papers that also consider a model of firm dynamics featuring default risk and aggregate fluctuations are Arellano et al. (2019) and Khan et al. (2021). The shared goal is to match the micro-evidence on the heterogeneous firm effects of the Great Financial Crisis as well as the path of aggregate variables. Arellano et al. (2019) study increases in firm uncertainty while Khan et al. (2021) consider the disruption in financial markets as driving forces. Both papers assume that the share of entrants always equals the share of firms which exit so that the total number of firms is constant. We depart from this assumption as our goal is to study whether financial frictions lead to inefficiencies in the number of operating firms.

Finally, the paper is related to a growing recent literature that studies earnings-based borrowing constraints. Empirically, Lian and Ma (2021) show that over 80% of US non-financial corporate debt is cash-flow based. Caglio et al. (2022) exploit new firm-bank matched data and show that earnings based constraints are even more important for SMEs. Ivashina et al. (2020) extend the analysis to Spain and Peru and conclude that cash-flow loans drive the contraction of credit supply during the Great Financial Crisis. On the theoretical front, Drechsel (2022) studies the macroeconomic effects of accounting for earnings-based constraints in a business cycle model. Our contribution to this literature is to provide a micro-foundation to cash-flow borrowing limits. To the extent to which current earnings are a good proxy for the present discounted value of all future earnings, the financial friction we propose here supports earnings-based borrowing constraints.

2 Empirical Evidence

Using granular empirical evidence, this section investigates the role of financial factors in firm exit decisions, and whether the relevance of these factors fluctuates with the business cycle. In order to do so, we first describe the sources of data used, we then explain the empirical strategy and finally present baseline results as well as robustness checks.

2.1 Data

The global company database ORBIS, produced by Bureau van Dijk, currently is the predominant source for multi-country firm-level analysis. Its coverage is particularly good for European countries as it collects data from a large number of smaller and medium-sized firms, which account for a substantial share of overall economic activity. Moreover, it reports annual balance sheet information and profit-loss statements, allowing us to measure the financial health of firms. This draft focuses on firms in the manufacturing sector in Italy, Portugal and the UK for the period 2002-2016.

The cleaning procedure is standard and relegated to the Appendix in the interest of space. Our final sample comprises 229,637 firms in Italy, 20,8637 in Portugal and 31,888 firms in the UK, which represent 59.0%, 63.2% and 6.0% of total output produced by the corresponding manufacturing sector according to EU Klems.[5] The sample closely resembles the actual firm size distribution of Italy and Portugal as determined by the share of total employment by size class, while small and medium-sized firms in the UK are under-represented in our sample (see Figure A.2). The key variable of interest is firm exit. We classify a firm as exiting at a time t whenever the firm reports any operation in period $t-1$ but no operation in period t. The average exit rate is 5.3% for Italy, 5.78% for Portugal and 7.18% for the UK. In comparison,

[5]In terms of employment the coverage is 62.0% (Italy), 70.0% (Portugal) and 8.4% (UK).

Eurostat reports average death rates of 5.5%, 9.7% and 8.7% correspondingly for the 2009-16 period.[6]

In order to minimize noise resulting from the level of disaggregation, the sample is divided into buckets based on firms' financial health. We first split firms in each industry-year pair into quintiles based on leverage, measured as the debt-to-assets ratio. We then take each of these five groups and classify them further into an additional set of quintiles according to the liquidity ratio. We are left with twenty-five equally sized groups of firms for each industry-year pair. The unit of observation is the group of firms in a given industry and year, as defined here.[7]

2.2 Empirical Model

The main focus of the regression analysis is firm exit – estimating how firm characteristics, the business cycle and the interaction of both affect the propensity to exit.

The role of financial factors The literature on firm exit has, so far, mostly focused on productivity as a predictor of firm survival. A novelty of this paper is the inclusion of firm-level financial information as part of the set of firm characteristics that determine firm exit. The first of the regression specifications studies whether financial health is a good predictor of firm exit, beyond firm fundamentals. More precisely:

$$Exit_{is,t} = \alpha FinHealth_{is,t-1} + X'_{is,t-1}\omega + \delta_s + \varphi_t + \sigma_{s,t} + \epsilon_{is,t} \tag{2.1}$$

where $Exit_{is,t}$ is the average exit rate in year t of firms belonging to group i in industry s. $FinHealth_{is,t-1}$ stands for a set of variables that measure the financial soundness of firms in group i during year *t-1*, $X'_{is,t-1}$ is a vector of non-financial factors and δ_s, φ_t and $\sigma_{s,t}$ are industry, year and industry-year fixed effects. Standard errors are clustered by industry-year, since some explanatory variables vary only at such level. The timing of the specification follows the theoretical literature such that the assumption is that future propensity to exit correlates with current firm characteristics.

The set of non-financial factors accounts for productivity as well as other firm fundamentals including age, size, output and labor growth and capital stock. In line with our model, the

[6]Structural Business Statistics by Eurostat provides data on business demography. Consistent with our definition of firm exit, we measure death rates as the ratio between the share of enterprise deaths in period t and the active population of enterprises in period t-1. The death rate for all three countries is only available since 2009. The average exit rate is our sample for the 2009-16 period is 5.5% for Italy, 6.36% for Portugal and 7.02% for the UK.

[7]As a robustness, we provide regression results at the firm level in the Appendix. The main results hold qualitative and quantitatively. The main difference is that, as expected, the R-squared is substantially lower when the unit of observation is the individual firm. Check Tables A.1 and A.2 for further details.

Table 1: Propensity to exit and financial factors

	UK			Italy			Portugal		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
debt to assets		0.051***			0.037***			0.038***	
		(0.002)			(0.004)			(0.007)	
liquidity ratio			-0.010***			-0.009***			-0.004***
			(0.001)			(0.001)			(0.001)
Groups	6,984	6,984	6,984	8,702	8,702	8,702	5,540	5,540	5,540
R^2	0.313	0.434	0.351	0.106	0.121	0.114	0.058	0.065	0.060
Industry-Year & Firm Size FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fundamentals	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The unit of observation is the industry-year bucket as defined in the main text. The dependent variable is the average exit rate. Firm fundamentals include the share of young firms (five years or younger), average labor productivity, average output and labor growth and the average capital stock. The regressions includes year, industry, industry-year and firm size fixed effects. Standard errors (in brackets) are clustered at the industry-year level.

baseline measure of productivity is labor productivity i.e. output per worker. To control for age, we build a dummy variable, young, that equals one if the firm is five years or younger. We then calculate the share of young firms in each group. To control for size, we construct firm size classes for each industry-year pair based on total assets. We then measure the share of firms in a group that belong to each class. We take operating revenue as our measure of output. We use the book value of fixed assets as capital stock and employment as labor input.

Our measures of financial soundness include leverage and liquidity ratios. The former is measured as the ratio of total debt to total assets while the latter is measured as the ratio of current assets to current debt.

Exit and the business cycle A key part of the empirical analysis studies how financial characteristics shape firm exit decision during recessions. To account for this, the next specification studies the interaction of financial factors with the business cycle:

$$Exit_{is,t} = \alpha FinHealth_{is,t-1} + X'_{is,t-1}\omega + \beta FinHealth_{is,t-1} \times \Delta u_t + X'_{is,t-1}\gamma \times \Delta u_t + \delta_s + \varphi_t + \sigma_{s,t} + \epsilon_{is,t} \quad (2.2)$$

where Δu_t indicates the change in the unemployment rate between *t-1* and *t*. We take fluctuations in unemployment as a proxy of the business cycle. While the coefficient of interest is β, the interaction between financial characteristics and unemployment, it must be interpreted jointly with α. The main hypothesis is that both coefficients feature the same sign.

2.3 Results

Table 1 reports results for the first regression specification. Columns (1), (4) and (7) capture the relationship between firm exit and firm fundamentals. Consistent with earlier studies, the

Table 2: Propensity to exit and financial factors over the business cycle

	UK			Italy			Portugal		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
debt to assets		0.051***	0.052***		0.037***	0.034***		0.038***	0.039***
		(0.002)	(0.002)		(0.004)	(0.004)		(0.007)	(0.007)
debt to assets x Δu			0.009***			0.015***			0.006*
			(0.002)			(0.004)			(0.003)
Groups	6,984	6,984	6,984	8,702	8,702	8,702	5,540	5,540	5,540
R^2	0.313	0.434	0.438	0.106	0.121	0.128	0.058	0.065	0.068
Industry-Year & Firm Size FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fundamentals	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The unit of observation is the industry-year bucket as defined in the main text. The dependent variable is the average exit rate. Firm fundamentals include the share of young firms (five years or younger), average labor productivity, average output and labor growth and the average capital stock. Δu_t is the change in the unemployment rates between $t-1$ and t. The regressions includes year, industry, industry-year and firm size fixed effects. Standard errors (in brackets) are clustered at the industry-year level.

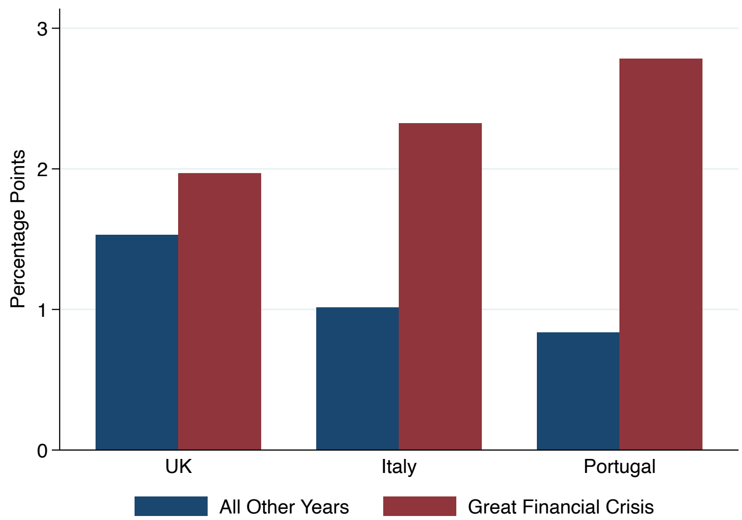
propensity to exit is significantly more prevalent among younger, smaller, less productive firms that experience smaller increases in output and factor growth and have lower holdings of capital than incumbents. While these findings are not new *per se*, they provide a useful benchmark against which we can compare how much predictive power financial factors add.

The rest of Table 1 focuses on two different measures of financial soundness, one at a time. We find that highly leverage firms have a higher propensity to exit, whereas firms with more liquidity are less likely to exit the following period. This holds for all three countries. Results are not only statistically significant but also economically relevant. A one standard deviation change in the debt to assets ratio delivers an increase of 1.8 (UK), 1.1 (Italy) and 1.3 (Portugal) in the propensity to exit in percentage points. This represents 15%, 24% and 31% higher exit rates than the average. A one standard deviation increase in the liquidity ratio delivers instead a reduction of 0.7 (UK), 0.6 (Italy) and 0.3 (Portugal) in the propensity to exit in percentage points. Given the difference in the magnitude of the coefficients as well as the corresponding R-squared, we take that leverage as our main empirical measure of financial health for the rest of the analysis.

A reasonable concern here is the following: suppose firm fundamentals, in particular firm productivity, are poorly measured. Moreover, assume firm productivity and firm leverage are negatively correlated. Then, the correlation reported in Table 1 is spurious. While the goal is to use the model to structurally identify the causal element of the correlation, a simple first pass is to check whether less productive firms indeed feature larger debt-to-assets ratios. Figure A.1 shows this does not fully hold. The relationship between productivity and leverage features a U-shape in all three countries: it is both highly productive and unproductive firms that take on more debt. In fact, the correlation between leverage and productivity is negative in the UK, but slightly positive in Italy and almost zero in Portugal.

Results for the second regression specification are summarized in Table 2. In particular, columns (3), (6) and (9) show how accounting for aggregate conditions affects the predictive

Figure 2: Differences in exit rates: normal times versus the Great Financial Crisis



Notes: This graph plots the estimated difference in propensity to exit between firms with debt-to-assets ratios one standard deviation apart. The regression model is given by

$$Exit_{is,t} = \alpha FinHealth_{is,t-1} + X'_{is,t-1}\omega + \beta FinHealth_{is,t-1} \times gfc_t + X'_{is,t-1}\gamma \times gfc_t + \delta_s + \varphi_t + \sigma_{s,t} + \epsilon_{is,t}$$

. The unit of observation is the industry-year bucket as defined in the main text. The dependent variable is the average exit rate. Firm fundamentals include the share of young firms (five years or younger), average labor productivity, average output and labor growth and the average capital stock. gfc_t is a dummy variable that equals one only for years 2008-2009. The regressions includes year, industry, industry-year and firm size fixed effects.

power of leverage. During bad times, defined as periods of positive unemployment growth, the correlation between leverage and exit is reinforced i.e. firms with high debt-to-assets ratios are even more likely to exit. In addition, the coefficients of the debt-to-assets ratio remain stable even as interaction terms are added.[8]

While this pattern holds in all three countries considered, the magnitude of the effect varies across them. To ease the comparison, Figure 2 plots the difference in the propensity to exit between firms that have debt-to-assets ratio one standard deviation in normal times versus during the Great Financial Crisis. Note that, in this case, the standard deviation is measured across all three countries and the regression model is a modified version of specification 2.2 where the business cycle indicator is gfc_t, a dummy variable that equals one in years 2008-2009, instead of Δu_t. Regression results are reported in the Appendix, Table A.4.

In the UK the difference in the propensity to exit jumps almost by 30% during the Great Financial Crisis from 1.5 to 1.9 percentage points. In Italy the likelihood of exit more than doubles, rising from 1 to 2.4 percentage points. Portugal experiences the largest increase of all three countries augmenting from 0.8 to 2.8 percentage points.

Overall, we take this as evidence that financial factors play a role in predicting firms' propen-

[8]As a robustness, Table A.3 provides results for an alternative measure of aggregate conditions: the log deviation of output from its long-term trend.

sity to exit and, thus, following our model, the prevalence of financial frictions in determining firm survival. The evidence here provided is not only qualitatively relevant, but quantitatively important. Moreover, the effect of these frictions is stronger during recessions suggesting that inefficient firm exit is counter-cyclical.

While the European results are encouraging, we expect to complement the analysis with novel data from the US Census in following drafts. We believe the data will improve the empirical analysis along four dimensions: (i) sample representativeness, providing a better match of the actual firm-size distribution; (ii) sample length, covering other downturns in addition to the Great Financial Crisis; (iii) accuracy of exit measure, by having full access to the universe of firms; and (iv) data frequency, in line with the business cycle convention that one period is one quarter.

3 Model

Next, we develop a dynamic model with firm dynamics and imperfect financial markets. In this section we present the environment, characterize agents' optimization problems and define the equilibrium. We discuss the role of market imperfections, as captured by a single parameter κ, and how this generates a wedge between efficient and competitive exit decisions.

3.1 Set up

3.1.1 Consumers

The representative household supplies its unit of labor inelastically at a price equal to one. It has preferences over consumption represented by the expected utility function

$$U = \sum_{t=0}^{\infty} \beta \ln \left(C_t \right),$$

where β is the discount factor. The aggregate consumption good is a Dixit-Stiglitz aggregator of differentiated varieties, $C_t = \left[\int_0^{J_t} c_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$, where c_{jt} represents consumption of variety j and J_t is the measure of available varieties, both at time t. Since each variety is produced by a single firm, J_t is also the measure of operating firms.

The household owns all firms in the economy. Thus, in addition to labor income, it receives net profits generated by production at time t. In order to smooth consumption, the household trades in risk-free bonds. We normalize the aggregate price index to 1, so that the inter-temporal optimization delivers a standard Euler equation:

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \right] \equiv \mathbb{E}_t \mathcal{M}_{t,t+1} \,,$$

where $\mathcal{M}_{t,t+1}$ is the stochastic discount factor.

3.1.2 Firms

Firms are monopolistically competitive. They operate a linear technology in labor and differ along their productivity levels, $y_{jt} = Z_t a_{jt} n_{jt}$. Z_t denotes aggregate total factor productivity at time t, a_{jt} denotes idiosyncratic productivity of firm j at time t, and n_{jt} denotes the amount of labor the firm is using in production. We assume that the idiosyncratic productivity component, a_{jt}, follows an AR(1) process in logs, with persistence parameter ρ_a, and normally distributed shocks with mean zero and standard deviation σ_a. We denote the conditional productivity distribution by $H(a_{jt+1}|a_{jt})$.

At the beginning of each period, an arbitrarily large mass $\mathcal{M}$ of potential entrants decide whether to pay an entry cost and enter the market. To enter the market, a potential entrant needs to make an irreversible investment before observing their productivity level. The cost of entry is idiosyncratic and uniformly distributed between 0 and $\mathcal{M} f_e$. Entrants observe their entry cost before making an entry decision, so that entry will follow a threshold rule. Using this formulation, the aggregate cost of entry is given by $\frac{1}{2} f_e M_e^2$, where M_e is the measure of entrants which pay the stochastic entry cost.[9]

Entry costs are denoted in effective units of labor, i.e., at time t an entrant that draws an entry cost x would need to pay $\frac{x w_t}{Z_t}$ to enter the market, where w_t is the real wage at time t. Upon paying the entry cost, entrants draw a productivity level from distribution $F(a)$, which we assume is log-normal distribution with mean $\underline{a}$ and standard deviation σ_a. At the same time, incumbents' current productivity is revealed.

Both incumbents and entrants are required to pay a fixed operating cost, f_{jt}, also in effective units of labor. Thus, the cost to firm j at time t is equal to $\frac{f_{jt} w_t}{a_{jt} Z_t}$. A firm can choose not to pay this cost and exit the market. The level of fixed costs, f_{jt}, is stochastic and follows an iid log-normal distribution, $G(f)$, with mean μ_f and standard deviation σ_f.

Firms have access to one-period debt securities with which they can pay the fixed operating

[9] To see this is the case, let the threshold level below which entrants pay the entry cost be $\bar{f}_e$. Then, total entry costs are

$$\mathcal{M} \int_0^{\bar{f}_e} \frac{x}{\mathcal{M} f_e} dx = \frac{1}{f_e} \left(\frac{\bar{f}_e^{\,2}}{2} \right) = \frac{1}{2} f_e M_e^2,$$

where $M_e \equiv \bar{f}_e / f_e$ is the endogenous measure of entrants that pay for entry. Note that $\mathcal{M}$ has no effect on equilibrium outcomes as long as it is large enough.

costs. We denote by b_{jt} the consumption goods firm j promises to repay in period $t+1$, in exchange for $q_{jt}b_{jt}$ today. We allow firm owners to renege on their previous debt obligations after observing their productivity levels. In the event of default, financial intermediaries take over the firm and the firm owner walks away with nothing. We restrict $b_{jt} > 0$, so that firms are not allowed to accumulate cash holdings.[10]

Conditional on operating, the firm sets prices and produces. Flow profits are given by

$$\pi_{jt} = \max_{\{p_{jt}, y_{jt}\}} \quad p_{jt}y_{jt} - \frac{y_{jt}w_t}{Z_t a_{jt}},$$
$$\text{s.t.} \quad y_{jt} = (p_{jt})^{-\epsilon} Y_t.$$

Solving the firm's problem, we obtain that profits are given by

$$\pi_t(a_{jt}) = \frac{1}{\epsilon} a_{jt}^{\epsilon-1} \overline{A}_t^{2-\epsilon} N_t Z_t, \tag{3.1}$$

where N_t is the measure of aggregate labor employed in direct production and $\overline{A}_t = \left(\int_0^{J_t} a_j^{\epsilon-1} dj\right)^{\frac{1}{\epsilon-1}}$ is a weighted average of idiosyncratic productivity.

At the end of the period, firms choose how much to cash out in the form of dividends. Dividends, however, are restricted to be weakly positive

$$d_{jt} = \pi_{jt} - \frac{f_{jt}w_t}{Z_t a_{jt}} - b_{jt-1} + q_{jt}b_{jt} \geq 0. \tag{3.2}$$

It is convenient for the recursive formulation to define repayment obligations at the beginning of period as the sum of debt holdings and fixed operating costs, $x_{jt} = b_{jt-1} + \frac{f_{jt}w_t}{Z_t a_{jt}}$. The idiosyncratic state of a firm is given by its current idiosyncratic productivity a_{jt}, and its repayment obligations x_{jt}, whereas the aggregate state Ω_t is given by the current aggregate productivity shock, Z_t, and the measure Λ_t of entrants and incumbents over productivity and debt levels. Since firms with the same idiosyncratic state make the same choices, we drop the index j for the rest of this section.

Prior to formally describing the firm's recursive problem, we provide a brief outline. The value of the firm is the discounted sum of its stream of future dividends. Every period, the firm chooses whether to default, how much to borrow and how many dividends to pay. It is subject to a budget constraint and a non-negative dividend condition.

If the firm can borrow to cover its financial obligations, it always chooses to do so. This option dominates defaulting as the non-negative dividend condition ensures the firm's value is

[10]The goal is to prevent firms from building a large buffer stock of unused credit to self-insure against idiosyncratic shocks. However, note that this assumption is also consistent with the data: the vast majority of firms have positive debt levels, after accounting for their cash holdings.

weakly positive. It then follows that the firm defaults only when its budget set is empty.

The Firm's Recursive Problem. To derive when firms default, let $\bar{L}(a_t;\Omega_t)$ be the maximal borrowing a firm with productivity a_t can raise when the aggregate state of the economy is Ω_t. The default decision boils down to whether the sum of profits and the borrowing limit suffices to cover the firm's repayment obligations. Let $\bar{f}_t = \bar{f}\,(a_t, b_{t-1};\Omega_t)$ be the highest fixed operating cost that ensures the non-negative dividend condition is satisfied. From condition (3.2), this threshold level is

$$\frac{\bar{f}_t w_t}{Z_t a_t} = \pi_t - b_{t-1} + \bar{L}(a_t;\Omega_t) \tag{3.3}$$

The firm chooses to repay if its fixed operating cost is lower than the corresponding threshold, $f_t \leq \bar{f}_t$.

We are now ready to consider the problem of an incumbent firm. Let $V\,(a_t, x_t;\Omega_t)$ denote the discounted value of the firm after all period t shocks are realized, denoted in period t final-good units. The value of the firm is

$$V\,(a_t, x_t;\Omega_t) = 0$$

for any state $(a_t, x_t;\Omega_t)$ such that the budget set is empty. In other words, whenever non-negative dividends are unfeasible despite borrowing the maximal amount, $d_t = \pi_t - x_t + \bar{L}(a_t;\Omega_t) < 0$. For all other states, firms choose new borrowing b_t and dividends d_t to solve

$$\begin{aligned} V\,(a_t, x_t;\Omega_t) = \max_{\{b_t,d_t\}} \; & d_t + \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\widetilde{V}\left(a_{t+1}, b_t + \frac{f_{t+1}w_{t+1}}{Z_{t+1}a_{t+1}};\Omega_{t+1}\right)\right], \\ \text{s.t.} \quad & d_t = \pi(a_t;\Omega_t) - x_t + q(a_t, b_t;\Omega_t)b_t\,, \\ & d_t \geq 0\,, \end{aligned} \tag{3.4}$$

where $\pi(a_t;\Omega_t)$ is given by equation (3.1), $q(a_t, b_t;\Omega_t)$ denotes the debt pricing schedule, which we derive in the next subsection, and $\mathcal{M}_{t,t+1}$ is the stochastic discount factor. The future value of the firm is given by

$$\widetilde{V}\,(a_{t+1}, x_{t+1};\Omega_{t+1}) = \begin{cases} V\,(a_{t+1}, x_{t+1};\Omega_{t+1}) & \text{if } f_{t+1} \leq \bar{f}_{t+1} \\ 0 & \text{otherwise} \end{cases}$$

3.1.3 Financial Sector

Firms borrow from financial intermediaries that act competitively and can diversify their lending so that they are not subject to idiosyncratic risk. This implies that the expected return on a loan to a firm should be equal to the real return on risk-free debt, $1 + r_t$.

In case of default, financial intermediaries can sell firms to new owners. Under new ownership, however, firms operate with productivity $(1-\kappa)a$, where a is the original productivity level. The parameter $\kappa \in [0,1]$ governs the degree of financial frictions.

In order for financial intermediaries to break even on lending, the debt pricing schedule solves the following equation,

$$
\begin{aligned}
q(a_t, b_t; \Omega_t) =& \mathbb{E}_t\left[\mathcal{M}_{t,t+1} G\left(\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right)\right)\right] \\
&+ \mathbb{E}_t\left[\mathcal{M}_{t,t+1} \frac{\widetilde{V}\left(\kappa a_{t+1}, \frac{f_{t+1} w_{t+1}}{Z_{t+1}\kappa a_{t+1}}; \Omega_{t+1}\right)}{b_t} \mathbb{1}\left[f_{t+1} > \bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right)\right]\right], \qquad (3.5)
\end{aligned}
$$

The first term captures the expected repayment value. Recall that the firm's default decision has a cutoff form. Repayment in period $t+1$ occurs as long as the fixed operating cost falls below the threshold $\bar{f}_{t+1}$, that is with probability $G(\bar{f}_{t+1})$. The second term captures the expected default value. Following default, financial intermediaries take over the firm and sell it to new owners. In exchange, financial intermediaries receive the value of the firm, $\widetilde{V}\left(\kappa a_{t+1}, \frac{f_{t+1} w_{t+1}}{Z_{t+1}\kappa a_{t+1}}; \Omega_{t+1}\right)$, that accounts for the productivity loss associated with new ownership and the lower repayment obligations implied by default.

Given the debt pricing schedule, the maximum financial intermediaries are willing to lend to a firm with productivity a_t when the aggregate state of the economy is Ω_t is

$$\bar{L}(a_t; \Omega_t) = \max_{b_t} q(a_t, b_t; \Omega_t) b_t.$$

3.2 Non-Stochastic Markov-Perfect Equilibrium

Let us now define the stationary non-stochastic steady state, in which $Z_t = 1$. For the equilibrium definition, we denote the joint distribution of firms across productivity levels and debt levels by $\Lambda(a, b)$. This measure includes entrants and incumbents prior to any financial decisions and prior to the realization of fixed operating costs. That is, it does not account for exit or changes in firm ownership and, thus, does not necessarily match the actual distribution of operating firms. Four components characterize the law of motion for this joint distribution: (i) the exit decision of firms, (ii) the borrowing decision of firms, (iii) the measure of entrants, and (iv) the exogenous law of motion for productivity. The law of motion for the joint distribution is defined as follows. For all Borel sets $\mathcal{A} \times \mathcal{B} \subset \mathbb{R}^+ \times \mathbb{R}$,

$$\begin{aligned}\Lambda'\left(\mathcal{A} \times \mathcal{B}\right) = &\int_f \int_{\mathscr{B}(a,b,f)} \int_{a' \in \mathcal{A}} dH(a'|a) d\Lambda(a,b) dG(f) \\ &+ \int_f \int_{\mathscr{B}(\kappa a,0,f)} \int_{a' \in \mathcal{A}} \mathbb{1}\left(f > \bar{f}(a,b)\right) dH(a'|\kappa a) d\Lambda(a,b) dG(f)\ , \\ &+ \mathbb{1}\left(0 \in \mathcal{B}\right) M_e' \int_{a' \in \mathcal{A}} dF(a')\ ,\end{aligned} \tag{3.6}$$

and

$$\mathscr{B}\left(a,b,f\right) = \left\{ (a,b,f) \quad \text{s.t.} \quad b'\left(a, b + \frac{fw}{a}\right) \in \mathcal{B} \right\},$$

where $b'(a,x)$ is the borrowing choice of a firm with productivity a and repayment obligations x. If a firm with state variables (a,x) chooses to default, then $b'(a,x) = \emptyset$. The first term in the transition function comes from firms that deliver on their debt obligations this period. The second term captures firms that default but operate under new ownership this period. The third term corresponds to next period's entrants. Note that these new firms enter the market with no debt.

In the stationary equilibrium, the distribution $\Lambda(a,b)$ is constant over time. The definition of the stationary non-stochastic Markov-perfect equilibrium is as follows:

Definition **1** (Equilibrium)**.** A stationary non-stochastic Markov-perfect equilibrium is a set of aggregate allocations $\{\bar{A}, N\}$, aggregate real wage w, a debt pricing schedule q, policy functions, a measure of entrants M_e and a distribution of firms over productivity and debt levels, $\Lambda(a,b)$, such that:

1. Borrowing and default policies solve the firm's problem.

2. Debt pricing policy solves equation (3.5).

3. Free entry condition holds:

$$f_e M_e w = \int_f \int_a \widetilde{V}\left(a, \frac{fw}{a}\right) dF(a) dG(f).$$

4. Labor market clears.

5. The level of labor employed in production, N, is given by

$$\begin{aligned}N = &\int \int \mathbb{1}\left(f \leq \bar{f}(a,b)\right) n(a) d\Lambda(a,b) dG(f) \\ &+ \int \int \mathbb{1}\left(f > \bar{f}(a,b)\right) \mathbb{1}\left(f < \bar{f}(\kappa a, 0)\right) n(\kappa a) d\Lambda(a,b) dG(f),\end{aligned}$$

where $n(a)$ is the policy function indicating how many workers a firm with productivity a hires for production. Note that this policy function is independent of b. The first term is the hiring of firms that repay their debt, and the second term is the employment of firms which default but continue operating.

6. Aggregate productivity satisfies its definition.

7. The distribution of firms is stationary.

3.3 The Role of κ

Canonical models of financial frictions assume that upon default, the lender can only recover a fraction of the collateral, most often capital. The lower the fraction, the higher the degree of financial frictions. Our model does not feature tangible capital, but considers intangible capital instead - the technology to produce and sell a differentiated good. When a firm defaults, the lender receives the firm's technology, which she can sell to a new owner. We assume that under new ownership, the productivity of the firm falls.[11] Instead of a_t, the productivity of the firm becomes κa_t, where $\kappa \leq 1$. This assumption captures the idea that the firm owner has some non-transferable knowledge on how to run the firm. Similar to models of financial frictions where a fraction of the capital stock vanishes upon default, here a fraction of the productive knowledge is lost. We start by characterizing how the degree of κ shapes the degree of inefficiency in the economy.

Firm dynamic models with exogenous labor supply are subject to two potential sources of inefficiency – one at the intensive margin and one at the extensive margin.[12] The intensive margin inefficiency arises when production inputs are misallocated across operating firms. The extensive margin inefficiency arises when there is a wedge between the efficient threshold of fixed operating costs that leads a firm to exit and the decentralized one.

We construct the model in a way that shuts down the intensive margin inefficiency - there is no misallocation of production since there is no markup dispersion.[13] The extensive margin inefficiency can be split further into two forces – a static and a, new, dynamic one. The static extensive margin inefficiency is discussed in Dhingra and Morrow (2019). If net profits differ from the social contribution of a firm to the economy, the rate of entry (and exit) is suboptimal. Since preferences are CES and fixed costs do not include intermediate inputs, our model does not feature this type of inefficiency. This is the case despite our assumption of convex aggregate

[11]The underlying assumption is that the original owner is best suited to run the firm. To the extent that these exist, firms would change ownership irrespective of default.

[12]When labor supply is endogenous, the aggregate markup in the economy can lead to an inefficient level of aggregate labor (see Edmond et al. (2015)).

[13]See Dhingra and Morrow (2019) and Edmond et al. (2015).

entry costs: given the stochastic nature of the individual cost, the marginal entrant pays exactly the marginal cost of entry. Thus, the only source of inefficiency in our model is an *extensive margin dynamic inefficiency*. Since financial markets are incomplete, the threshold of fixed operating costs that pushes a firm to default can be lower in the decentralized equilibrium relative to the first-best allocation.

The social planner would like the firm to pay its fixed operating cost as long as it falls below the discounted sum of current and future profits. This is because profits fully represent the social contribution of the firm. However, the threshold that pushes firms into default and potentially towards exit in the decentralized equilibrium can be lower than the efficient threshold level.

The wedge between the decentralized threshold and the efficient one is the result of two forces. First, the firm owner may have outstanding debt obligations. In contrast to the firm owner, the social planner does not take debt into account when deciding whether a firm should default. This is because debt repayment is a transfer between agents. Second, the borrowing limit faced by the firm can be lower than the discounted sum of its expected future net profits.

The degree of κ governs the size of the extensive dynamic inefficiency. The lower is κ, the more inefficient the decentralized equilibrium allocation relative to the first-best one. In the proposition below, we show that when $\kappa = 1$ the decentralized equilibrium allocation is identical to the first-best allocation.

PROPOSITION **1.** *When $\kappa = 1$ the decentralized equilibrium allocation is identical to the first-best one.*

We relegate the formal definition of the first-best allocation to the Appendix. Despite the lack of complete markets, when κ is equal to 1 the decentralized equilibrium is efficient. The proof of Proposition 1 shows that in the case of $\kappa = 1$, the borrowing limit of firms is equal to their expected discounted net profits stream. Since default does not affect firm productivity when $\kappa = 1$, the amount of debt obligations of a firm does not affect its probability of exit. A high level of debt can result in default, but the firm can continue operation under new ownership. As a result, the threshold for firm exit is identical to the efficient one.

Recall that financial markets in our model are incomplete even when $\kappa = 1$. The debt issued by the firm is non-state-contingent if the firm repays its debt. Despite market incompleteness, the firm can raise funds equal to its future discounted net-profits stream. To raise these level of funds, a firm needs to issue enough debt so that its default probability equals to 1. The level of debt issuance effectively acts as equity financing because creditors understand they will obtain the firm in the following period. Even though debt is non-state-contingent upon repayment, the returns to lenders are fully state-contingent as they will obtain the firm in the following period without any loss in productivity when $\kappa = 1$.

Table 3: Calibration

Parameter	Description	Value	Target/Source	Data	Model
(a) Externally calibrated parameters					
β	Firm discount factor	0.9	Annual frequency		
r^f	Risk free rate	0.03	$\beta^m = 0.97 > \beta$		
ϵ	Elasticity of substitution	4	Nakamura and Steinsson (2010)		
σ_a	Productivity shock s.d.	0.13	Gopinath et al. (2017)		
ρ_a	Productivity persistence	0.59	Gopinath et al. (2017)		
(b) Internally calibrated parameters					
$\underline{a}$	Entrants average productivity	-0.8	Employ. share of young firms	0.088	0.093
μ_f	Fixed operating cost mean	-1.835	Exit rate	0.076	0.066
σ_f	Fixed operating cost s.d.	0.8	Exit rate of young firms	0.130	0.167
κ	Recovery rate	0.7	Log(debt) coefficient	0.021	0.045

Notes: Young firms are firms that have been operating for four years or less. Log(debt) coefficient refers to the WLS estimator of the following regression specification: $exit = \beta log(debt) + \delta_a + \epsilon$.

4 Quantitative Analysis

In this section we quantify the cost of financial frictions due to firm exit. To do so, we first calibrate the stationary equilibrium of the model to match key features of the US economy. We characterize the model's stationary firm distribution and simulate the economy following a recessionary shock. We then compare annual consumption in our baseline economy relative to a world without financial frictions.

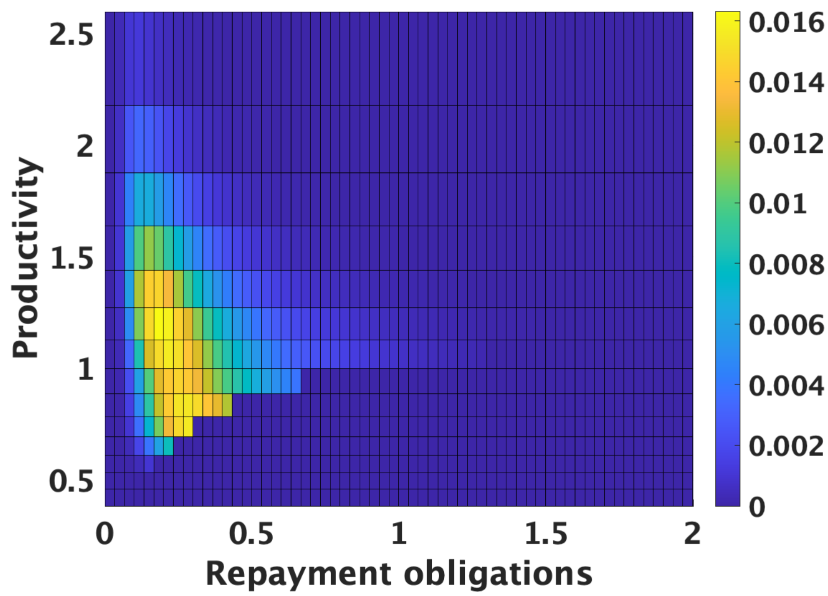
4.1 Calibration

We calibrate the model at an annual frequency to a combination of US Census Bureau data and the European ORBIS data used in section 2 as well as standard values from the literature. There are ten structural parameters in the model. We set the entry cost, f_e, to 0.03 such that the number of entrants is normalized to one without loss of generality.

Table 3 summarizes the choice of the remaining parameters, which can be divided into two groups. For the first group of five parameters, we borrow values from existing papers. We set the firm discount factor, β, to 0.9 while setting the market discount factor, β^m, to 0.97. The latter implies a risk free rate equal to 3%. We set the elasticity of substitution, ϵ, to 4 as in Nakamura and Steinsson (2010). For the parameters that govern the firm productivity process, (ρ_a, σ_a) we borrow Gopinath et al. (2017) estimates, which use Spanish manufacturing data from ORBIS.

A second group of four parameters is internally calibrated: the two parameters that shape the distribution of the fixed operating cost, μ_f and σ_f; entrants average productivity, $\underline{a}$; and the recovery rate κ. We choose these parameters to match the following empirical moments: (i) the share of employment of young firms, (ii) the overall exit rate, (iii) the exit rate among young firms, and (iv) the correlation between exit and debt. For the first three moments, we

Figure 3: Ergodic Distribution



Notes: This figure displays the ergodic distribution of firms along the two state variables: productivity and the size of repayment obligations.

use the Business Dynamics Statistic dataset. Young firms are firms that have been operating for four years or less.

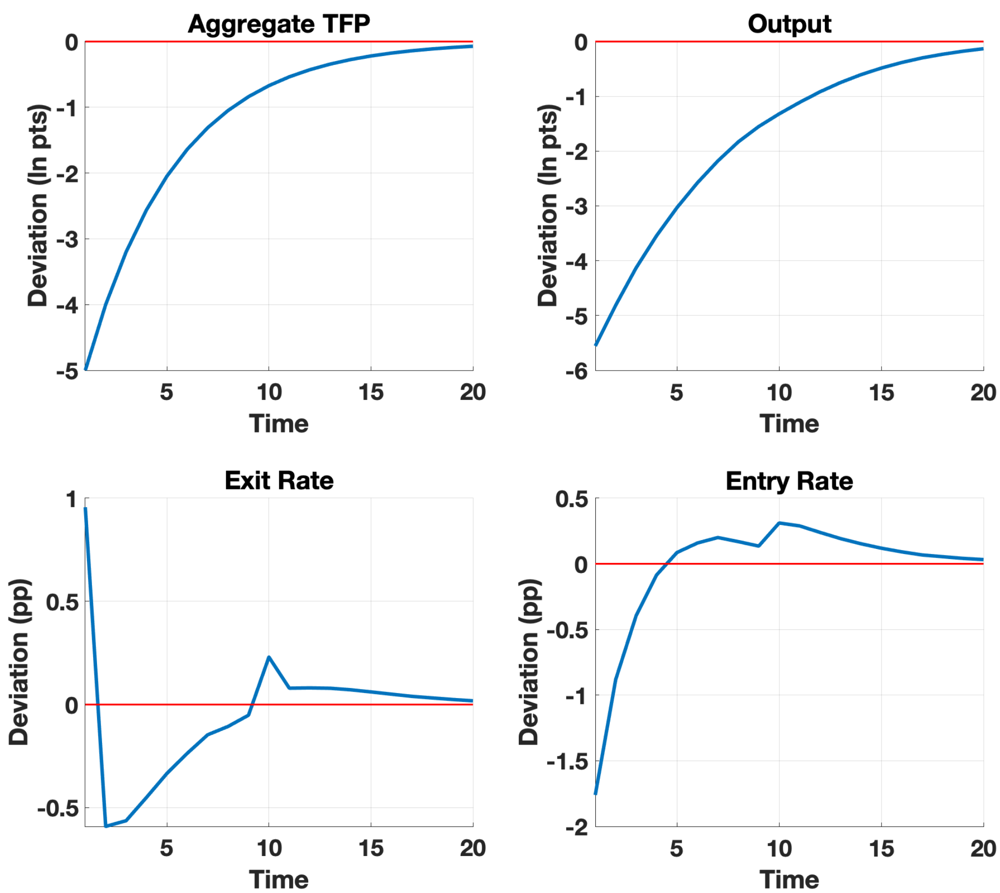
The last moment is estimated using the ORBIS dataset. We run an regression specification that resembles the empirical model given by 2.1. However, in order to be consistent with the model, instead of leverage the key regressor is debt in logs, while we control for total assets also in logs. The moment reported in table 3 is the WLS estimator. As a theoretical counterpart, we regress exit on debt in logs controlling for productivity grid points.

4.2 Features of the Calibrated Economy

Figure 3 displays the ergodic distribution of firms along productivity levels and repayment obligations. Most of the state space, in dark blue, features no firms, as they choose to exit the market. Instead, firms are concentrated along the productivity level equal to one, given our calibration of the productivity process. Firms with a history of large operating costs move towards the right, where repayment obligations are larger. At the same time, entrants are concentrated towards the origin, as they feature lower productivity levels but also start with zero debt, thus, facing lower repayment obligations.

Next, we turn to study the response of the economy to a negative TFP shock. We posit that Z_t follows an AR(1) process in logs with persistence parameter 0.8. We analyze the response of the economy to an unexpected negative TFP shock that reduces aggregate productivity by

Figure 4: IRF to a recessionary shock



Notes: This figure displays the response of the economy to an unexpected negative TFP shock.

5% on impact. Figure 4 displays the results.

The top right panel of Figure 4 displays the response of output. On impact, the decline in output is very similar to the decline in aggregate TFP. The bottom left panel reports the response of exit rate. On impact exit rate goes up by 1 percentage point. The bottom right panel of the figure displays the response of the entry rate, in percentage points deviation from its steady state rate.

The endogeneity of both entry and exit imply that the output losses due to the recession build up over time. 10 periods following the shock, aggregate TFP is less than 1% lower than its steady state level, while the output loss is greater than 1%.

Finally, we run a similar regression to the one in Section 2 upon the impact of the shock. We find that during the recession the estimated coefficient of debt on exit goes up by 31.3%. That is, our model is able to endogenously deliver a stronger relationship between debt and exit during a recession despite the fundamental degree of financial frictions, κ, being constant over time.

4.3 The Costs of Financial Frictions

In our calibrated example, firms exit 6.6% of the time while the default rate is 11.1%. This is a world with a productivity loss of 30% new firm ownership. In order to evaluate the cost of financial frictions, we simulate the same economy under zero productivity loss i.e. $\kappa = 1$. As previously argued, this corresponds to the first best provided the economy starts from the steady state. Results show that, unsurprisingly, there is far less exit. In particular, the measure of firms increases by 27.5%. More importantly, annual consumption is 2.1% higher, implying that the welfare costs in this example are sizable.

5 Government Intervention

[Work in progress]

The decentralized equilibrium allocation is constrained efficient. That is, taking the debt pricing condition as given, the government cannot improve the market allocation without providing liquidity to firms. In this section, we study how direct government intervention can help alleviate the costs of financial frictions.

While unlimited resources allow the government to restore the first-best allocation in models without firm exit, this is not the case in our model as subsidies give rise to zombie firms.[14] There is a trade-off between the prevention of inefficient firm exit at the expense of keeping unproductive firms operating. This section will analyze how policy tools should balance these two opposing forces.

6 Conclusion

This paper explores the effect of financial frictions on firm exit. Using European firm level data, we provide suggestive evidence that there is a strong relationship between firm leverage and propensity to exit. Moreover, we find that this relationship strengthens during recessions. In order to quantify the cost of financial frictions due to firm exit, we propose a model of firm dynamics and defaultable debt with no other sources of misallocation. We find that welfare losses are sizable: annual consumption is 2.1% lower relative to first best. We then study the role of government interventions.

[14] We refer to zombie firms as firms that manage to operate thanks to a subsidy but the social planner would prefer to shut down.

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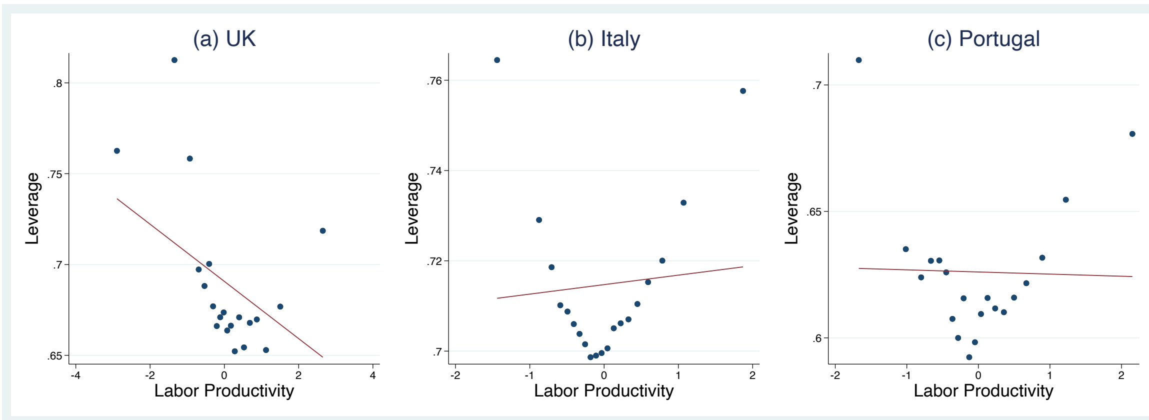
A Additional Figures

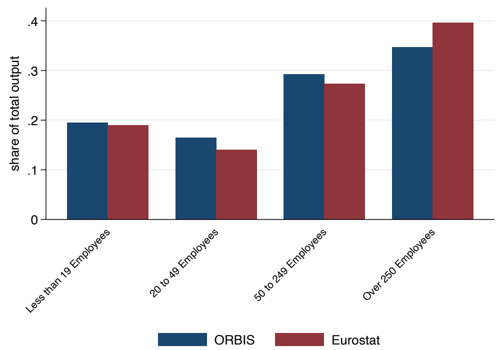
Figure A.1: Firm Exit and Financial Health



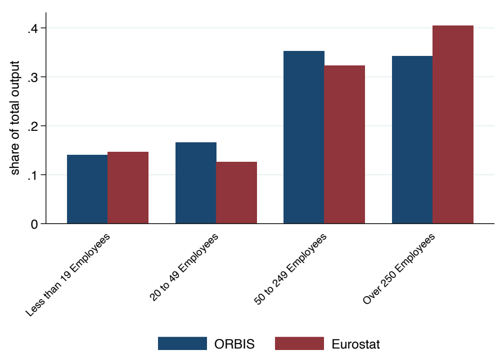
Notes: This graph plots the relationship between leverage and productivity, controlling for year and industry fixed effects. Leverage is measured by the debt to assets ratio. Productivity is the log deviation of firm labor productivity from its industry-year average. Firm-year observations are grouped into bins to ease visualization. The source of the data is Orbis

Figure A.2: Firm size distribution

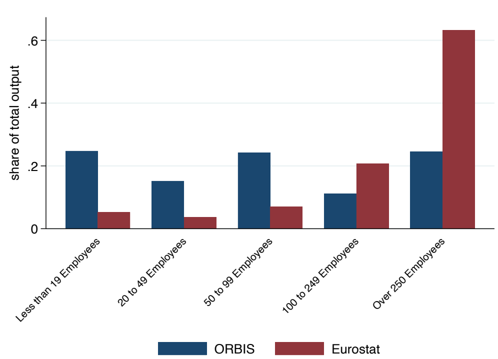
Panel A: Italy



(b) Panel B: Portugal



(c) Panel C: UK



Notes: This figure plots the fraction of total employment accounted for by firms belonging to each size class. The blue bars report statistics from our sample and the red bar from Eurostat.

B Additional Tables

Table A.1: Propensity to exit and financial factors - at the firm level

	UK			Italy			Portugal		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
debt to assets		0.018***			0.050***			0.048***	
		(0.002)			(0.003)			(0.007)	
liquidity			-0.006***			-0.009***			-0.004***
			(0.001)			(0.001)			(0.001)
N	31,888	31,888	31,888	229,637	229,637	229,637	20,861	20,861	20,861
R^2	0.032	0.035	0.034	0.008	0.012	0.009	0.019	0.026	0.020
Year, Industry, Industry-Year & Firm Size FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fundamentals	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The unit of observation is the firm. The dependent variable is $exit_t$, a dummy variable that equals one when a firm that operates in year $t-1$, shows no operation in period t. Firm fundamentals include the indicator $young$ (=1 if the firm is five years or younger), labor productivity, output and labor growth and the capital stock. The regressions includes year, industry, industry-year and firm size fixed effects. Standard errors (in brackets) are clustered at the industry-year level.

Table A.2: Propensity to exit and financial factors over the business cycle - at the firm level

	UK			Italy			Portugal		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
debt to assets		0.018***	0.017***		0.050***	0.046***		0.048***	0.050***
		(0.002)	(0.002)		(0.003)	(0.003)		(0.007)	(0.007)
debt to assets x Δ u			0.008*			0.019***			0.009**
			(0.003)			(0.003)			(0.004)
N	31,888	31,888	31,888	229,637	229,637	229,637	20,861	20,861	20,861
R^2	0.032	0.035	0.035	0.008	0.012	0.013	0.019	0.026	0.027
Year, Industry, Industry-Year & Firm Size FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fundamentals	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The unit of observation is the firm. The dependent variable is $exit_t$, a dummy variable that equals one when a firm that operates in year $t-1$, shows no operation in period t. Firm fundamentals include the indicator $young$ (=1 if the firm is five years or younger), labor productivity, output and labor growth and the capital stock. Δu_t is the change in the unemployment rates between $t-1$ and t. The regressions includes year, industry, industry-year and firm size fixed effects. Standard errors (in brackets) are clustered at the industry-year level.

Table A.3: Propensity to exit and financial factors over the business cycle - cyclical component of output

	UK		Italy		Portugal	
	(1)	(2)	(3)	(4)	(5)	(6)
debt to assets	0.049***	0.049***	0.025***	0.024***	0.038***	0.036***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.006)	(0.006)
debt to assets x cycle		-0.445***		-0.155***		-0.172
		(0.140)		(0.054)		(0.156)
Groups	6,984	6,984	8,702	8,702	5,540	5,540
R^2	0.642	0.645	0.161	0.167	0.075	0.076
Year, Industry, Industry-Year & Firm Size FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fundamentals	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The unit of observation is the industry-year bucket as defined in the main text. The dependent variable is the average exit rate. Firm fundamentals include the share of young firms (five years or younger), average labor productivity, average output and labor growth and the average capital stock. $cycle_t$ is the log deviation of output from its long-term trend. The regressions includes year, industry, industry-year and firm size fixed effects. Standard errors (in brackets) are clustered at the industry-year level.

Table A.4: Propensity to exit and financial factors over the business cycle - the Great Financial Crisis

	UK		Italy		Portugal	
	(1)	(2)	(3)	(4)	(5)	(6)
debt to assets	0.049***	0.047***	0.025***	0.022***	0.038***	0.033***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.006)	(0.006)
debt to assets x gfc		0.017***		0.022**		0.024*
		(0.004)		(0.009)		(0.014)
Groups	6,984	6,984	8,702	8,702	5,540	5,540
R^2	0.642	0.647	0.161	0.169	0.075	0.077
Year, Industry, Industry-Year & Firm Size FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fundamentals	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The unit of observation is the industry-year bucket as defined in the main text. The dependent variable is the average exit rate. Firm fundamentals include the share of young firms (five years or younger), average labor productivity, average output and labor growth and the average capital stock. gfc_t is a dummy variable equal to one in 2008 and 2009. The regressions includes year, industry, industry-year and firm size fixed effects. Standard errors (in brackets) are clustered at the industry-year level.

C First-best allocation and proof of Proposition 1

We start by showing the optimal decision rules of the social planner. First, we show that static firm profits in the decentralized equilibrium are equal to their overall contribution to welfare. A special case of the result in Dhingra and Morrow (2019). Then, we derive the socially optimal level of exit and entry.

C.1 Efficient allocation across operating firms

Consider the problem of a planner, trying to maximize aggregate consumption (C) by allocating N units of labor across a measure J of heterogeneous productivity firms. Recall the productivity of firm j is the product of their individual productivity (a_j) and aggregate productivity (Z). This sub-problem of the planner is as follows:

$$\begin{aligned} \max & \left(\int_0^J c_j^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \\ \text{s.t.} \quad & \int_0^J \frac{c_j}{a_j Z} dj = N. \end{aligned}$$

Taking first-order conditions, we obtain that for all j

$$c_j^{-\frac{1}{\epsilon}} C^{\frac{1}{\epsilon}} = \frac{1}{a_j Z}\lambda. \tag{C.1}$$

By dividing the optimality conditions of two different firms, we obtain

$$\frac{c_j}{c_k} = \left(\frac{a_k}{a_j}\right)^{-\epsilon}.$$

Substituting into the resource constraint we have

$$\frac{1}{Z} c_j a_j^{-\epsilon} \int_0^J a_k^{\epsilon-1} dk = N. \tag{C.2}$$

Let $\overline{A} \equiv \left(\int_0^J a_j^{\epsilon-1} dj\right)^{\frac{1}{\epsilon-1}}$, so that we have

$$c_j = a_j^{\epsilon} \overline{A}^{1-\epsilon} Z N. \tag{C.3}$$

Plugging into the definition of aggregate consumption, we obtain

$$C = \left(\int_0^J \left(a_j^{\epsilon} \overline{A}^{1-\epsilon} ZN \right)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} = \overline{A}^{1-\epsilon} ZN \left(\int_0^J a_j^{\epsilon-1} dj \right)^{\frac{\epsilon}{\epsilon-1}} = \overline{A} ZN. \tag{C.4}$$

C.2 Efficient firm entry and exit

Next, we turn to the problem of the planner of choosing which firms should exit and how many firms should enter. The planner needs to choose which firms operate, taking into account how each firm raises aggregate productivity but lowers the amount of labor that is used for direct production. Recall the definition of aggregate productivity is given by

$$\overline{A}_t = \left(\int_0^{J_t} a_{jt}^{\epsilon-1} dj \right)^{\frac{1}{\epsilon-1}}.$$

Flow benefits of operation. The contribution of firm j to aggregate productivity is given by

$$\int_0^{a_j} \frac{\partial \overline{A}}{\partial a_{jt}} da_{jt} = \frac{1}{\epsilon-1} \overline{A}_t^{2-\epsilon} a_{jt}^{\epsilon-1}, \tag{C.5}$$

and therefore the contribution to output of firm j, gross of the labor for fixed operating costs, is given by

$$\frac{1}{\epsilon-1} \overline{A}_t^{2-\epsilon} Z_t N_t a_{jt}^{\epsilon-1}, \tag{C.6}$$

Flow costs of operation in terms of output. The decline in output due to the operating cost of a firm with productivity a_t and operating costs f_t is equal to

$$-\overline{A}_t \frac{f_t}{a_t}. \tag{C.7}$$

The recursive value of a firm. Let $W_{FB}(a, f; \Omega_t)$ denote the value of a firm with productivity a and operating cost f which operates at time t:

$$W_{FB}\left(a_t, f_t; \Omega_t\right) = \left(\frac{1}{\epsilon-1} \overline{A}_t^{2-\epsilon} Z_t N_t a_t^{\epsilon-1} - \overline{A}_t \frac{f_t}{a_t} \right) \frac{1}{\overline{A}_t Z_t N_t} + \beta \mathbb{E}_t \left[\max \left\{ W_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right), 0 \right\} \right]. \tag{C.8}$$

Let $\tilde{W}_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right) \equiv \max\left\{ W_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right), 0 \right\}$ be the social value of the firm before deciding whether to pay the fixed operating costs. Rearranging:

$$W_{FB}\left(a_t, f_t; \Omega_t\right) = \left(\frac{1}{\epsilon-1} \overline{A}_t^{1-\epsilon} Z_t N_t a_t^{\epsilon-1} - \frac{f_t}{a_t} \right) \frac{1}{Z_t N_t} + \beta \mathbb{E}_t \left[\tilde{W}_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right) \right]. \tag{C.9}$$

The social planner chooses to continue operation if and only if $W_{FB}(a_t, f_t; \Omega_t) \geq 0$, which boils down to

$$\frac{f_t}{a_t Z_t} \leq \frac{1}{\epsilon - 1} \overline{A}_t^{1-\epsilon} N_t a_t^{\epsilon-1} + \beta N_t \mathbb{E}_t \left[\tilde{W}_{FB}(a_{t+1}, f_{t+1}; \Omega_{t+1}) \right]. \tag{C.10}$$

So we define

$$\overline{f}(a_t; \Omega_t) \equiv a_t Z_t \left[\frac{1}{\epsilon - 1} \overline{A}_t^{1-\epsilon} N_t a_t^{\epsilon-1} + \beta N_t \mathbb{E}_t \left[\tilde{W}_{FB}(a_{t+1}, f_{t+1}; \Omega_{t+1}) \right] \right]. \tag{C.11}$$

When f_t is greater than this threshold, the planner would decide not to pay the operating cost of the firm, and the firm would exit. If its below the threshold, the firm continues operating.

Finally, recall that the total entry cost is given by $\frac{1}{2} f_e M_e^2$. Therefore, the marginal cost of an additional entrant is $f_e M_e$. As a result, the utility cost of paying the entry cost for the marginal entrant is given by

$$\frac{f_e M_e}{Z_t N_t}.$$

Thus, the planner would want more firms to enter until

$$\frac{f_e M_e}{Z_t N_t} = \int \int \tilde{W}_{FB}(a_t, f_t; \Omega_t) dF(a_t) dG(f_t). \tag{C.12}$$

C.3 Decentralized equilibrium when $\kappa = 1$

Proof of Proposition 1. To prove that the decentralized equilibrium coincides with the first-best one, we will show that the equilibrium conditions in the decentralized economy are equivalent to the optimality conditions of the social planner. In particular, we will show that aggregate consumption in the decentralized equilibrium is equal to (C.4), the exit threshold is given by (C.11), and free entry satisfies (C.12).

The static problem of an operating firm is

$$\pi_{jt} = \max_{\{p_{jt}, y_{jt}\}} \quad p_{jt} y_{jt} - \frac{y_{jt} w_t}{Z_t a_{jt}},$$
$$\text{s.t.} \quad y_{jt} = (p_{jt})^{-\epsilon} Y_t.$$

Solving for the firm's optimal price

$$p_{jt} = \frac{\epsilon}{\epsilon - 1} \frac{w_t}{Z_t a_{jt}}$$

By solving the cost minimization problem of households, we can obtain that the aggregate price index is given by $P_t = \left(\int p_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$. Plugging the expressions for p_j and using our normalization $P_t = 1$, we obtain

$$\frac{\epsilon}{\epsilon - 1}\frac{w_t}{Z_t}\frac{1}{\overline{A}_t} = 1.$$

Rearranging this equation pins down the real wage in the economy

$$w_t = \frac{\epsilon - 1}{\epsilon} Z_t \overline{A}_t.$$

Plugging back into the pricing decision of firms, we have $p_{jt} = \frac{\overline{A}_t}{a_{jt}}$. Thus, given the production function, we can write labor demand as

$$n_{jt} = \frac{a_{jt}^{\epsilon-1}}{\overline{A}_t}\frac{Y_t}{Z_t}$$

Aggregating across firms delivers:

$$Y_t = \overline{A}_t Z_t N_t. \tag{C.13}$$

where N_t is the aggregate level of labor employed directly in production. Note that this is identical to (C.4). That is, there is no misallocation across operating firms as the markup is constant across firms.

The operating profits of firm j at time t are therefore given by

$$\pi(a_{jt}; \Omega_t) = \frac{1}{\epsilon} a_{jt}^{\epsilon-1} \overline{A}_t^{2-\epsilon} N_t Z_t. \tag{C.14}$$

Recall that the borrowing limit is given by

$$\overline{L}(a_{jt}; \Omega_t) = \max_b q(a_t, b; \Omega_t) b, \tag{C.15}$$

where

$$\begin{aligned} q(a_t, b_t; \Omega_t) =& \mathbb{E}_t \left[\mathcal{M}_{t,t+1} G \left(\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) \right) \right] \\ &+ \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \frac{\widetilde{V}\left(\kappa a_{t+1}, \frac{f_{t+1} w_{t+1}}{Z_{t+1} \kappa a_{t+1}}; \Omega_{t+1} \right)}{b_t} \mathbb{1}\left[f_{t+1} > \bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) \right] \right]. \end{aligned} \tag{C.16}$$

Define the borrowing amount when debt issuance is equal to b_t to be $L(a_{jt}, b_t; \Omega_t)$. So that $\overline{L}(a_{jt}; \Omega_t) = \max_b L(a_{jt}, b; \Omega_t)$. Using the equation above, when $\kappa = 1$ we have

$$\begin{aligned} L(a_{jt}, b_t; \Omega_t) =& \mathbb{E}_t \left[\mathcal{M}_{t,t+1} G \left(\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) \right) b_t \right] \\ &+ \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \widetilde{V} \left(a_{t+1}, \frac{f_{t+1} w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1} \right) \mathbb{1}\left[f_{t+1} > \bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) \right] \right]. \end{aligned} \tag{C.17}$$

We start by showing that the borrowing amount is weakly increasing in b. Differentiating with respect to b_t we obtain

$$\frac{\partial L(a_{jt}, b_t; \Omega_t)}{\partial b_t} = \mathbb{E}_t \left[\mathcal{M}_{t,t+1} G\left(\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right)\right)\right]$$
$$+ \mathbb{E}_t \left[\mathcal{M}_{t,t+1} g\left(\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right)\right) \frac{Z_{t+1} a_{t+1}}{w_{t+1}} \left[\tilde{V}\left(a_{t+1}, \frac{\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right) - b_t\right]\right], \tag{C.18}$$

where we've used $\frac{\partial \bar{f}(a_{t+1}, b_t; \Omega_{t+1})}{\partial b_t} = -\frac{Z_{t+1} a_{t+1}}{w_{t+1}}$. From the equation above, we can see that if

$$\tilde{V}\left(a_{t+1}, \frac{\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right) \geq b_t, \tag{C.19}$$

for all a_{t+1} and Ω_{t+1}, then $\frac{\partial L(a_{jt}, b_t; \Omega_t)}{\partial b_t}$ is positive. By definition, we have

$$\tilde{V}\left(a_{t+1}, b_t + \frac{\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right) \geq 0, \tag{C.20}$$

because the firm chooses not to default. Note that since the firm chooses not to default $\tilde{V}(\cdot) = V(\cdot)$ which is defined in (3.4):

$$\begin{aligned} V\left(a_t, x_t; \Omega_t\right) = \max_{\{b_t, d_t\}} & \; d_t + \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \tilde{V}\left(a_{t+1}, b_t + \frac{f_{t+1} w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right)\right], \\ \text{s.t.} & \quad d_t = \pi(a_t; \Omega_t) - x_t + q(a_t, b_t; \Omega_t) b_t \\ & \quad d_t \geq 0 \end{aligned} \tag{C.21}$$

Using the envelope theorem, we have that

$$\frac{V\left(a_t, x_t; \Omega_t\right)}{\partial x_t} = -\lambda, \tag{C.22}$$

where λ is the Lagrange multiplier on the dividend constraint. The FOC with respect to d_t imply that

$$\lambda = 1 + \nu,$$

where $\nu \geq 0$ is the Lagrange multiplier on the non-negativity of dividends constraint. So we obtain $\frac{V(a_t,x_t;\Omega_t)}{\partial x_t} \leq -1$ for all $\{a_t, x_t, \Omega_t\}$. We can now use this derivative to obtain

$$
\begin{aligned}
&\widetilde{V}\left(a_{t+1}, \frac{\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right) \\
&= V\left(a_{t+1}, \frac{\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right) \\
&= V\left(a_{t+1}, b_t + \frac{\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right) - \int_0^{b_t} \frac{\partial V\left(a_{t+1}, b + \frac{\bar{f}(a_{t+1}, b_t; \Omega_{t+1}) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right)}{\partial x_t} db \\
&\geq V\left(a_{t+1}, b_t + \frac{\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right) + b_t \\
&\geq b_t
\end{aligned}
$$

where the second-to-last inequality follows from $\frac{V(a_t,x_t;\Omega_t)}{\partial x_t} \leq -1$ and the last inequality follows from (C.20). Thus, condition (C.19) is satisfied and $\frac{\partial L(a_{jt},b_t;\Omega_t)}{\partial b_t} \geq 0$.

As a result, we have that $\overline{L}(a_{jt}; \Omega_t) = \lim_{b_t \to \infty} L(a_{jt}, b_t; \Omega_t)$. Plugging into the $L(\cdot)$ function, we obtain

$$
\overline{L}(a_{jt}; \Omega_t) = \mathbb{E}_t\left[\mathcal{M}_{t,t+1} \widetilde{V}\left(a_{t+1}, \frac{f_{t+1} w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right)\right] \tag{C.23}
$$

because $\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right) < 0$ when $b_t \to \infty$. Note that when $\kappa = 1$, the firm exits if and only if $f > \bar{f}\left(a_{t+1}, 0; \Omega_{t+1}\right)$. This is because after default, the productivity of the firm remains unchanged. And the fact that $\bar{f}\left(a_{t+1}, b_t; \Omega_{t+1}\right)$ is decreasing in b_t. So if the firm owner would choose to exit without any debt, the firm would also choose to default with any level of positive debt. So using equation (3.3), we have that the exit threshold is

$$
\bar{f}(a_t; \Omega_t) = a_t Z_t \left[\frac{\pi(a_t; \Omega_t)}{w_t} + \frac{1}{w_t} \bar{L}(a_t; \Omega_t)\right]. \tag{C.24}
$$

Plugging the expression for profits and the real wage we obtain

$$
\bar{f}(a_t; \Omega_t) = a_t Z_t \left[\frac{1}{\epsilon - 1} a_{jt}^{\epsilon - 1} \overline{A}_t^{1-\epsilon} N_t + \frac{\epsilon}{\epsilon - 1} \frac{1}{Z_t \overline{A}_t} \bar{L}(a_t; \Omega_t)\right]. \tag{C.25}
$$

To show that the threshold rule coincides with that of the planner, we need to show that

$$
\bar{L}(a_t; \Omega_t) = \overline{A}_t Z_t N_t \frac{\epsilon - 1}{\epsilon} \beta \mathbb{E}_t\left[\tilde{W}_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right)\right].
$$

Using the expression for $\bar{L}(a_t; \Omega_t)$, this condition becomes

$$
\mathbb{E}_t\left[\mathcal{M}_{t,t+1} \widetilde{V}\left(a_{t+1}, \frac{f_{t+1} w_{t+1}}{Z_{t+1} a_{t+1}}; \Omega_{t+1}\right)\right] = \overline{A}_t Z_t N_t \frac{\epsilon - 1}{\epsilon} \beta \mathbb{E}_t\left[\tilde{W}_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right)\right].
$$

Using the stochastic discount factor this condition becomes

$$\mathbb{E}_t\left[\frac{\widetilde{V}\left(a_{t+1}, \frac{f_{t+1}w_{t+1}}{Z_{t+1}a_{t+1}}; \Omega_{t+1}\right)}{\overline{A}_{t+1}Z_{t+1}N_{t+1}}\right] = \mathbb{E}_t\left[\frac{\epsilon-1}{\epsilon}\tilde{W}_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right)\right]. \tag{C.26}$$

We will show that

$$\frac{\epsilon}{\epsilon-1}\frac{\widetilde{V}\left(a_{t+1}, \frac{f_{t+1}w_{t+1}}{Z_{t+1}a_{t+1}}; \Omega_{t+1}\right)}{\overline{A}_{t+1}Z_{t+1}N_{t+1}} = \tilde{W}_{FB}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right) \tag{C.27}$$

when $\kappa = 1$, so the condition (C.26) is satisfied.

To show this is the case, we *conjecture* that when $\kappa = 1$ the firm can maximize its value function by taking $b_t \to \infty$. That is, effectively promising to give the firm to creditors in the following period. In that case, we have that

$$V\left(a_t, x_t; \Omega_t\right) = \pi(a_t; \Omega_t) - x_t + \bar{L}(a_t; \Omega_t). \tag{C.28}$$

Using the expression for the borrowing limit we obtain

$$V\left(a_t, x_t; \Omega_t\right) = \pi(a_t; \Omega_t) - x_t + \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\widetilde{V}\left(a_{t+1}, \frac{f_{t+1}w_{t+1}}{Z_{t+1}a_{t+1}}; \Omega_{t+1}\right)\right]. \tag{C.29}$$

Let's define $W(a_t, f_t; \Omega_t) \equiv \frac{\epsilon}{\epsilon-1}\frac{V\left(a_t, \frac{f_t w_t}{Z_t a_t}; \Omega_t\right)}{\overline{A}_t Z_t N_t}$. By defining $W(\cdot)$ in this way, to show that condition (C.26) is satisfied, it is sufficient to show that $W(\cdot) = W_{FB}(\cdot)$. Using equation (C.29) we obtain

$$W\left(a_t, f_t; \Omega_t\right) = \frac{\epsilon}{\epsilon-1}\frac{1}{Z_t\overline{A}_t N_t}\left(\pi(a_t; \Omega_t) - \frac{f_t w_t}{Z_t a_t} + \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\widetilde{V}\left(a_{t+1}, \frac{f_{t+1}w_{t+1}}{Z_{t+1}a_{t+1}}; \Omega_{t+1}\right)\right]\right), \tag{C.30}$$

Using

$$\widetilde{V}\left(a_{t+1}, \frac{f_{t+1}w_{t+1}}{Z_{t+1}a_{t+1}}; \Omega_{t+1}\right) = \frac{\epsilon-1}{\epsilon}\overline{A}_{t+1}Z_{t+1}N_{t+1}\widetilde{W}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right),$$

where $\widetilde{W}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right) \equiv \max\left\{W\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right), 0\right\}$, and the definition of the stochastic discount factor, we obtain

$$W\left(a_t, f_t; \Omega_t\right) = \frac{\epsilon}{\epsilon-1}\frac{1}{Z_t\overline{A}_t N_t}\left(\pi(a_t; \Omega_t) - \frac{f_t w_t}{Z_t a_t}\right) + \beta\mathbb{E}_t\left[\widetilde{W}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right)\right], \tag{C.31}$$

Using the equilibrium level of the wage as well as that of profits we obtain

$$W\left(a_t, f_t; \Omega_t\right) = \left(\frac{1}{\epsilon-1}\overline{A}_t^{1-\epsilon} Z_t N_t a_t^{\epsilon-1} - \frac{f_t}{a_t}\right)\frac{1}{Z_t N_t} + \beta \mathbb{E}_t\left[\widetilde{W}\left(a_{t+1}, f_{t+1}; \Omega_{t+1}\right)\right], \tag{C.32}$$

Note that this is equivalent to equation (C.9). Thus, condition (C.26) is satisfied. We confirm our conjecture that taking $b_t \to \infty$ indeed maximizes the owner's value by noting that they obtain the complete-markets value of the firm. Therefore, the exit thresholds in the decentralized equilibrium are identical to those in the first-best allocation.

Finally, we turn to show that the level entry is identical to the first-best allocation. The free-entry condition in the decentralized equilibrium is

$$\frac{f_e M_e w_t}{Z_t} = \int\int \widetilde{V}(a_t, \frac{f_t w_t}{Z_t a_t}; \Omega_t) dF(a_t) dG(f_t). \tag{C.33}$$

Using the definition of w_t we obtain

$$\frac{f_e M_e}{Z_t} = \int\int \frac{\epsilon}{\epsilon-1}\frac{1}{Z_t \overline{A}_t}\widetilde{V}(a_t, \frac{f_t w_t}{Z_t a_t}; \Omega_t) dF(a_t) dG(f_t). \tag{C.34}$$

And then using the definition of $\tilde{W}(\cdot)$ we have

$$\frac{f_e M_e}{Z_t N_t} = \int\int \tilde{W}(a_t, f_t; \Omega_t) dF(a_t) dG(f_t). \tag{C.35}$$

This condition is equivalent to (C.12) as $\tilde{W}(a_t, f_t; \Omega_t) = \tilde{W}_{FB}(a_t, f_t; \Omega_t)$. Therefore, the decentralized free-entry condition coincides with that in the planner's problem. As all optimality conditions in the decentralized equilibrium are identical to those of the planner, we conclude that when $\kappa = 1$ the market allocation is identical to the first-best one.

■