How do central banks control inflation? A guide for the perplexed*

Laura Castillo-Martinez
Duke University

Ricardo Reis
LSE

August 2023

Abstract

Central banks have a primary task of pursuing price stability. They do so by issuing different forms of money, setting an array of interest rates, and managing their balance sheets. This article surveys the economic theories that justify the central bank's ability to use these tools to control inflation around a target and applies them to interpret historical fluctuations in inflation. It presents alternative approaches as consistent with each other, as opposed to conflicting ideological camps. Each of them relies on equilibrium forces in different markets within a common dynamic general equilibrium structure.

E31, E52, E61.
Keywords: monetary policy, policy rules, determinacy, effectiveness.

*Contacts: l.castillo-martinez@duke.edu and r.a.reis@lse.ac.uk. We are grateful to students at the LSE, as well as at the AEA continuing education program, Columbia University, the Kiel Institute, and the Tinbergen Institute for many comments on this material, to Maxi Guenneweig and Saleem Bahaj for co-teaching it with us, to Martina Fazio, Jose Alberto Ferreira, and Edoardo Leonardi for editorial support, and to the editor, referees, and many colleagues over the years for their comments. This project received funding from the European Union's Horizon 2020 research and innovation programme, INFL, under grant number No. GA: 682288.

How do central banks keep inflation on target? How do they prevent episodes of hyperinflation and their tragic consequences for welfare? Can the central bank control inflation if the economy goes through secular stagnation, a liquidity trap, or a fiscal crisis? Why did inflation rise in 2021-... and will it persist? These are crucial questions that have answers in current economic theory.

Yet, students coming out of a macroeconomics class are often flummoxed by this topic. Undergraduates mostly retain that central banks print money and more money means higher inflation. They are then thoroughly confused when they realize that most central banks barely mention money in their speeches, that they do not actually choose how much money to print, and that the US monetary base increased five-fold between 2008 and 2014 with no visible dent on inflation. Graduate students learn about the setting of interest rates and the Phillips curve, and perhaps even about the welfare costs of inflation and the links between monetary and fiscal policy. However, as soon as you ask them to reconcile the Fisher principle (higher interest rates are associated with higher expected inflation one-to-one), the Taylor rule (increasing interest rates more than one-to-one in response to inflation keeps inflation constant), and the empirical evidence (exogenous positive shocks to interest rates lower inflation) you are likely to get an incoherent answer. Discussions on equilibrium determinacy or active-passive regimes attract theoretically-minded researchers as much as they put off those focused on empirical applications.

The goal of this article is to provide a unified treatment of the theory of how central banks control inflation. The hope is that researchers will have an accessible entry point to this literature, so they can make sense of monetary policies and inflation outcomes. Our approach has three distinctive features. The first is that we highlight the common features of different viewpoints by using a single general-equilibrium model of the economy. We present alternative theories less as opposing views and more as focussing on different equations and markets within the same model that exert different forces over inflation. The second feature is that we put the central bank at the center of all of them. It is the central bank whose liabilities define the price level and who has a mandate to target a value for it. The central bank's policies are always the key determinant of inflation, whether they can be described as monetary, fiscal or, more accurately, as a mix of both. The third feature is that we provide one interpretation of the recent history of inflation as a result of different strategies followed at different times. Our goal is not to defend it as the only or the right explanation for inflation's movements, but rather to let the reader see the different theoretical concepts at work and their relative strengths.

This is not an article about the optimal way to conduct monetary policy or about how to trade off variability in inflation versus real activity. We take as given a target for inflation, and study how the central bank goes about delivering actual inflation as close as possible to this target. This involves three dimensions of evaluating policy. The first is a *determinacy* question, on whether policy can deliver a unique price level. We take multiple or indeterminate equilibria in models with rational expectations as signs of an incomplete policy framework. The second is an *effectiveness* question, on whether one policy leads to a smaller variation in the deviations of actual inflation from target. By characterizing the components of this variation, these can be quantified to make the choice between policies based on predicted outcomes. The third is *experience*, as we compare different inflation episodes of both advanced economies and emerging markets and link them to the monetary policy regimes of the time.

Section 1 sets up the canonical dynamic model we will use and shows that the classical analysis of supply and demand does not pin down inflation. This is what makes this topic special. To discuss inflation, one must introduce a central bank. This section lays out the tools at the central bank's disposal, and sets up a passive strategy where they are not used and inflation remains indeterminate. The sections that follow activate one tool at a time. A tool is associated with an approach, or strategy, to control inflation. Each approach focuses on a particular market, is associated with a specific equilibrium condition, and moves the price level through a distinct economic force. Central banks choose rules that link the tool to exogenous policy choices and the most effective rule provides one way to compare the approaches.

In section 2, we consider the approach that focuses on financial markets and the forces of arbitrage that price financial assets. The key equilibrium condition is the Fisher equation, but determinacy relies also on a terminal condition that depends on the rationality of expectations and on the credibility of regime switches. The policy tool is the interest rate that remunerates bank deposits at the central bank, and the associated rules are feedback rules, like the Taylor rule. The effectiveness of this policy strategy relies on keeping the expectations of savers close to those of the central bank, which puts transparency, communication, and the management of these expectations at the center. The inflation targeting regimes that dominated monetary policy in the 1990s and 2000s testify to the success of this approach, even though testing it econometrically is challenging.

Section 3 continues with setting interest rates, but in the version that dominated the actions of the major central banks in the 2010s (and earlier in the case of Japan). Concerns about lower bounds on the rate justified policies of forward guidance, going long, or subsidized bank credit. These put a great burden on the rationality of the economic agents interacting with the central bank, as their effectiveness relies on how far-sighted they are in setting their expectations. The experience with these strategies has been mixed.

Section 4 looks at the money market and how the price level may adjust to ensure it is in equilibrium. The key equation is the demand for currency, while the policy tool is the central bank's exclusive right to supply banknotes. We consider money growth rules, fiscal rules on the seignorage revenues from this activity, and pegs to either commodities like gold or to foreign currencies. Determinacy is easy to ensure, but the theory suggests that this policy strategy is usually not effective. The experience using money growth rules in the early 1980s in the UK and US and with the pegs of the Bretton Woods system in the post-war confirms that the monetary approach often leads to volatile inflation. The experience in Latin America in the 1980s and 1990s shows that it can even come hand in hand with hyperinflation.

In section 5, the key equation is an intertemporal budget constraint. The approach in this section relies on the solvency of the central bank as the price level adjusts to reflect changes in the value of the central bank's liabilities. This approach is usually called a fiscal-theory of price level determination, but it applies to a central bank that uses the size and composition of its balance sheet and the resulting fluctuations in its net income as the main tool. In practice, this strategy is more often imposed on, rather than adopted by, central banks when they lose their financial independence.

Arbitrage in financial markets, currency and payments, and solvency of the institution whose liabilities define the unit of account, are three separate economic forces that co-exist in a well-specified model and interact with each other. Interacting with all of them are nominal rigidities, and the equilibrium in goods markets where firms set prices in nominal units. Because this force is so central, we discuss it in every section and emphasize how it interacts with the others.

Section 6 discusses how a coherent policy framework must choose one strategy to be active and the others to be passive but all consistent with each other, requiring that one policy tool is dominant over the others. It concludes with a brief discussion of the inflation disaster of 2021-... and how central banks might have let it happen.[1]

[1]Previous surveys, taking a different approach, are McCallum (1999) and Woodford (2003).

1 Inflation in equilibrium

We start with a neoclassical model that will frame the discussion. The world starts at date 0, and we state the key equilibrium equations and refer the reader to the online appendix where they are micro-founded.

1.1 A classical economy

Starting with *savers and investment markets*, the key equation is an Euler equation that has the form:

$$\mathbb{E}_t\left[M_{t+1}(1+R_t)\right] = 1. \tag{1}$$

The $\mathbb{E}_t(.)$ operator captures the expectations of the private sector as of date t. Savers may not have full information, but we will assume they are rational in the sense of being consistent with the other equilibrium equations of the model.[2] The R_t is the promised return at date t on a real safe investment that costs one unit of goods today and pays R_t in units of goods at date $t+1$. The M_{t+1} is a stochastic discount factor. Intuitively, M_{t+1} reveals how many units of a good private agents would require next period in exchange for one unit of the good today. In other words, M_{t+1} is the marginal rate of substitution between consumption today and tomorrow. Since $1+R_t$ is the opportunity cost of consuming one more unit today in terms of foregone consumption tomorrow, the equation above states that agents must be indifferent between consuming or saving an extra unit. [3]

Moving to *consumers and the goods market*, households also equate marginal rates of substitution and relative prices but now across goods within the same period. Letting $\Re(i)$ be how many units of good i consumers would trade for one unit of good 0, and $P(i)$ be the nominal price of good i, then the conditions are:

$$\Re_t(i) = \frac{P_t(i)}{P_t(0)} \quad \text{for } i = 1, ..., I, \tag{2}$$

one for each good of which there are I.

[2]Section 3 will relax this assumption where it especially matters, when forming far-away expectations.

[3]An alternative, but equivalent, investment intuition is that to ensure no-arbitrage profits, it must be that the risk and time adjusted net return on any investment is zero. The stochastic discount factor provides the adjustment factor for time and risk. If investors are risk neutral then M_{t+1} would be equal to a constant β that captures solely impatience, and the equation states that the real return is approximately equal to the rate of time preference $-\ln(\beta)$.

Turning to *firms, workers and the labor market*, they maximize the surplus from production. Their interaction delivers a desired price for each good that equals a markup over the marginal cost of production. Letting the desired price be $\tilde{P}_t(i)$, the markup $Z_t(i)$, and the real marginal cost of production be a function $F(Y_t(i), Q_t)$ that depends on how much is produced $Y_t(i)$ and on the real cost of inputs, Q_t:

$$\tilde{P}_t(i) = Z_t(i) P_t F(Y_t(i), Q_t) \,. \tag{3}$$

A common setup is to assume that each good is produced by a monopolist firm that faces a consumer demand function with a constant price elasticity and uses only labor under constant returns to scale. In that case $Z_t(i)$ is a constant and the real cost of inputs is the real wage.

Next is the definition of the price index over which we will measure inflation. It is an index of individual prices:

$$P_t = \mathbb{P}\left(\{P_t(i)\}_{i=0,\dots I}\right) \,, \tag{4}$$

that is linearly homogeneous so that it doubles when all prices double. It is a cost-of-living index if it is the dual of the consumption aggregator $C_t = \mathbb{C}\left(\{C_t(i)\}_{i=1,\dots I}\right)$, over which households get utility $\mathbb{U}(C_t)$ that they discount by a subjective factor β.

Finally, are the set of market clearing conditions that close the system. We will stick to an economy without savings domestically or abroad, so consumption equals income for each good $C_t(i) = Y_t(i)$. The intertemporal marginal rate of substitution is: $M_{t+1} = \beta \mathbb{U}'(Y_{t+1}) / \mathbb{U}'(Y_t)$ while the static marginal rate of substitution is: $\Re(i) \equiv \frac{\partial \mathbb{C}(.)/\partial C_t(i)}{\partial \mathbb{C}(.)/\partial C_t(0)}$. The marginal rate of substitution between consumption and labor depends on the disutility from supplying labor according to the utility function $\mathbb{V}(.)$ that depends on how much is produced, so: $Q_t = \mathbb{V}'(Y_t)/\mathbb{U}'(Y_t)$.

There are two missing pieces in this economy: a mapping from desired prices $\tilde{P}_t(i)$ to actual prices $P_t(i)$, and a central bank. Before introducing them, we can formalize the starting point of indeterminacy that makes this topic special.

1.2 Price level (in)determinacy

Our goal in this paper is to study the sequence $\{P_t\}_{t=0}^{\infty}$ and the associated inflation sequence $\Pi = \{\Pi_t\}_{t=1}^{\infty}$ where $\Pi_t \equiv P_t / P_{t-1}$.

Definition 1. *The level of inflation is unique or determinate in equilibrium if:*

1. *There is a unique scalar P_0 in equilibrium.*

2. *If Π' and Π'' both satisfy equilibrium conditions, then $\Pi' = \Pi''$.*

The first condition requires that even if the entire future path of inflation is pinned down from today onwards, one must still know today's price level. What ultimately matters is how much is a dollar's worth in terms of real goods at any given date. Without pinning down the initial value of a dollar, for a given inflation path, the actual price level P_t could be any number. The second condition states that inflation must be unique.

If actual and desired prices are the same, $\tilde{P}_t(i) = P_t(i)$, then we can already solve for the real outcomes in equilibrium in the classical model. Aggregating across goods using equations (3) and (4) gives a solution for output. This also pins down marginal rates of substitution, and so the solution for the real interest rate and relative prices in equations (1) and (2).

The key result is that $\Re_t(i)$ and M_t are both exogenous with respect to P_t. This is commonly referred to as the classical dichotomy: real trade-offs are unchanged regardless of the price level.

Without a central bank, there is nothing to pin down the price level. Any level of inflation is consistent with the equations above describing the real equilibrium in the economy. This result dates back to Hume (1752): dollars are just a unit of account with which the prices of goods are determined. If people started denominating prices in cents instead of dollars nothing would change. There is no demand or supply that ensures that 100 cents equals one dollar. Nothing in classical economics pins down the price level or inflation, in the same way that nothing determines whether measurements should be in inches or centimeters.

1.3 Nominal rigidities

There are many ways to break the classical dichotomy. In this paper, we will generally focus on, arguably, the most popular one: nominal rigidities that drive a wedge between desired and actual prices. Moreover, we stick to one particular model of nominal rigidities, the Calvo price-setting model. However, each section will also discuss alternative ways of breaking the classical dichotomy and their impact on inflation.

Introducing nominal rigidities requires log-linearizing the economy around a steady state classical equilibrium where, given the equations above, the real interest rate and inflation are equal to constants, β and $\bar{\Pi}$. Using small letters to denote the log-linearized counterpart of capital letters, the savers' log-linearized equilibrium condition is:

$$y_t - \mathbb{E}_t(y_{t+1}) + \theta r_t = 0, \tag{5}$$

where θ is a fixed parameter capturing the curvature of M_{t+1} with respect to output. This comes from equation (1) together with market clearing in the goods market.

The desired price equation in (2)-(4) is replaced by a Phillips curve:

$$\pi_t = \beta \mathbb{E}_t(\pi_{t+1}) + \kappa\alpha(y_t - y_t^n) + z_t. \tag{6}$$

The new variable is y_t^n, the natural level of output, which is the solution under the classical dichotomy as derived above. The parameter α summarizes the curvature of both the marginal cost function and the disutility of labor supply, while κ is a parameter that captures the inverse of the degree of nominal rigidities, so that when $\kappa \to \infty$ we are back at the classical dichotomy.

These equations still do not pin down inflation: there are now two equations in three unknowns, r_t, y_t and π_t. Worse, the indeterminacy of the price level is also an indeterminacy of output and all other real variables. For any given path for inflation, firms and workers produce whatever is demanded at these prices. In turn, demand dictates an amount of savings that determines the real interest rate. Other models of nominal rigidities will have different mechanisms, but share this joint nominal and real indeterminacy.[4]

1.4 The central bank

In modern digital economies, people use electronic means of payment, like debit and credit cards, to settle their transactions. For any given transaction, the seller may have an account in bank A while the buyer has an account in bank B, so there must be a settlement whereby bank A collects payments from bank B. The central bank is the clearing house where payments between banks take place. These payments are made using another digital mean of payment, often named reserves. Reserves are nothing but liabilities of the central bank vis-à-vis banks. Because reserves are the ultimate form of payment, they are the unit of account of the economy. Since reserves in the United States are denominated in

[4] Carlstrom and Fuerst (2002) and Nakajima and Polemarchakis (2005) provide further discussion on sticky prices and real indeterminacy.

dollars, firms and people choose to denominate their prices in dollars as well. The price of a good is simply how many units of reserves must be exchanged to obtain such good.

The current stock of reserves is a list of entries in a spreadsheet at the central bank, one for each bank. Given its control over the spreadsheet, the central bank has two ways to set monetary policy: it can choose the amount of reserves, V_t, or the rate at which it remunerates them, I_t^v. These are nothing but decisions on the sum of the entries in the spreadsheet at any point in time, and on the rate at which the number in each entry of the spreadsheet rises across periods. A minimal central bank is simply the manager of this spreadsheet.

Central banks have for decades used the spreadsheet to fix the interest rate of one-period nominal bonds, I_t. In the past, this was done indirectly by rationing the supply of reserves, and then buying and selling bonds for reserves to target this interest rate. Since the great financial crisis, reserves have been abundant so they are a pure financial asset that provides no payment or liquidity services. In that case, no arbitrage between nominal bonds and reserves delivers $I_t = I_t^v$.

The optimality condition of households with respect to nominal bonds is:

$$\mathbb{E}_t \left[M_{t+1} \left(\frac{(1 + I_t) P_t}{P_{t+1}} \right) \right] = 1. \tag{7}$$

The intuition is the same as the one that applied to real assets: as reserves promise a nominal interest rate I_t, their real return depends on inflation. Indifference towards holding them must result from equating this expected return times the marginal rate of substitution between consumption today and tomorrow to one.

Real-life central banks do more than manage their spreadsheet. For one, they issue banknotes and commit to exchange them for reserves one for one at all times. Banknotes are distinct from reserves in five ways. First, they can be freely held by anyone in the economy, not just banks. Second, they are physical and the central bank can produce them at close-to-zero cost. Third, they are anonymous as people do not have to declare to the government how much currency they have or from whom they got it. Fourth, for some payments it may be easier to use banknotes than electronic means backed by reserves (while for others the opposite is true). Fifth, banknotes pay no interest.

The first four properties create a demand for the services provided by banknotes separate to the demand for reserves. Some economic agents prefer not to use the banking system when making payments, some prefer physical to digital payments, some want anonymity in their transactions, and some find cash easier to use. We capture these preferences with a utility function: $\mathbb{H}(H_t/P_t)$ where $H_t \geq 0$ are the banknotes held in nominal units. The fifth property implies that the opportunity cost of using banknotes as opposed to the digital means of payment provided by the banking sector is the interest rate paid on reserves. Therefore, at an optimum the marginal rate of substitution between banknotes and consumption must equal this opportunity cost:

$$\frac{\mathbb{H}'(H_t/P_t)}{\mathbb{U}'(C_t)} = \frac{I_t}{1+I_t}. \tag{8}$$

When the central bank prints currency, it can get goods from agents in return. This gives rise to a resource flow called seignorage. Since it costs close to nothing to produce currency and there is a downward-sloping demand for it, currency is not a liability of the central bank, but rather a durable good that it produces and sells for its value $1/P_t$. Seignorage is $S_t^H = (H_t - H_{t-1})/P_t$, and the central bank could rebate it right away to the government as a dividend D_t. However, central banks also have expenses E_t and they use their reserves to buy a portfolio of assets with current real value A_t and a risky return $(1 + R^a_{t+1})$. Therefore, their total net surplus S_t ends up being:

$$S_t = S_t^H - D_t - E_t + (1 + R_t^a - (1 + I_{t-1})P_{t-1}/P_t)A_{t-1}. \tag{9}$$

The central bank's net worth is the difference between its assets and its liabilities: $W_t = A_t - V_t/P_t$. The central bank cannot be running a Ponzi scheme, since on the other side are the private agents who would not want to give away their resources for free. Therefore, this net worth must equal the present value of its expected deficits:

$$\left(\frac{(1+I_{t-1})P_{t-1}}{P_t}\right) W_{t-1} = -\,\mathbb{E}_t\left(\sum_{j=0}^{\infty} M_{t,t+j}S_{t+j}\right). \tag{10}$$

Operationally, this points to an extra tool that the central bank can use to control inflation: the size and composition of its balance sheet determining its surpluses.

To sum up, a central bank is the manager of a spreadsheet of payments on reserves, a seller of an infinitely-lived durable good in currency, and a borrower and lender from the private sector through its balance sheet. Say that: (i) the central bank sets the interest rate on reserves passively to satisfy equation (7) for whatever inflation may be, (ii) currency either has no value $\mathbb{H}'(.) = 0$ or the central bank passively prints it to satisfy whatever demand comes from equation (8), and (iii) the central bank holds no assets $A_t = 0$ and pays as dividends its seignorage minus expenses so that surplus $S_t = 0$. Then, the equations above are redundant. They just pin down what the endogenous I_t, H_t will be. Inflation continues to be indeterminate.

Yet, in exercising each of these three functions, the central bank has tools to affect the price level through each of the equations above. Each section that follows will focus on an approach to monetary policy that relies on each one of these three tools. From this benchmark of indeterminacy, each section will separately relax one of (i) to (iii) while keeping the others passive.

Before that, no model of inflation is complete without specifying how the central bank interacts with the fiscal authority. For now, we assume that the government collects its own taxes to pay for its spending in excess of the dividends from the central bank. This is sometimes called a Ricardian policy, or that fiscal policy is passive. In section 5, we will introduce government bonds and discuss alternatives. Having introduced central bank assets and durable goods, there is an also an extra optimality condition from the household that must be stated. At infinity the utility value of the wealth held by the consumer must be zero, otherwise she would be better off consuming more and saving less. This is the transversality condition:

$$\lim_{T\to\infty} M_{t,T}\left(\frac{H_T + V_T}{P_T}\right) = 0. \tag{11}$$

1.5 The policy target

The policy goal of the central bank is to keep P_t close to a target P_t^* at all dates. The target may be stochastic, have a unit root, or depend on the real state of the economy. It may be arbitrary or optimal given some objectives of policy.[5] The key assumption is that it is exogenous with respect to P_t.

Policy rules describe how the central bank fixes its tool. We denote the exogenous choice by X_t^i, to highlight that it is exogenous with respect to P_t, and with the superscript denoting the tool it refers to. The policy rule may also imply a feedback from the actual price level to the policy tool, in which case the policy rule is a map from P_t and X_t^i to the tool, in this example, I_t.

The policy information of the central bank is limited by imperfect real-time estimates

[5] Readers interested in the choice of P_t^* can see Khan, King and Wolman (2003) or Woodford (2010).

of the state of the economy or of the desired inflation target. While $\mathbb{E}_t(p_{t+j})$ denoted the public's expectation at t of what the log price level will be at date $t+j$, we use $\hat{p}_{t+j}$ to denote the central bank's expectation at t, and these may not be the same.

The effectiveness of a policy is assessed by the size of the deviations between the log price level and its target :

$$\varepsilon_t \equiv p_t - p_t^*. \tag{12}$$

The central bank wants to set its tool following a rule that makes its expectation of these errors zero: $\hat{p}_t = \hat{p}_t^*$. We call this the *most effective* rule, and denote its choice by X_t^*.

2 The no-arbitrage approach: setting interest rates

Combining equations (1) and (7) gives a no-arbitrage relation between real and nominal bonds:

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{1+I_t}{\Pi_{t+1}} \right) \right] = 0. \tag{13}$$

This states that, once adjusted by the stochastic discount factor, savings in real investment or in reserves at the central banks must yield the same expected return. It is often called the Fisher equation and it is the key equation of the approach in this section.

The Fisher equation captures an economic force that can move the price level. It works as follows: banks can choose to hold reserves or real investments. Suppose the price level today was too low. Relative to a fixed future price level, then expected inflation would be higher. Therefore, the return on reserves would be lower than the return on real investments. In other words, by holding reserves at the central bank, banks get fewer goods in return than if they had invested them privately. Banks would want to hold zero reserves and invest all of their resources in real terms, which would not be an equilibrium given a positive supply of reserves. Rather, as banks demand fewer reserves, their value falls. Because reserves are the unit of account, their real value is $1/P_t$, so the price level must rise back into equilibrium. A higher price level means that expected inflation is lower and the real return on reserves is higher, rising until the point where banks are, once again, indifferent between real investment and reserves.[6]

[6] A small literature has studied inflation using the no-arbitrage approach but when incomplete markets lead to variations of equation (13), see Benassy (2000) and Den Haan, Rendahl and Riegler (2017).

2.1 Exogenous interest rate rules

By itself, relying on arbitrage and setting interest rates does not determine inflation: it depends on how it is done.

2.1.1 Interest rate pegs

If the central bank chooses the interest on reserves exogenously, then $I_t = X_t^i$. The literature has traditionally referred to this as an interest rate peg. The Fisher equation then implies that:

$$\mathbb{E}_t \left(\frac{M_{t+1}}{\Pi_{t+1}} \right) = \frac{1}{1 + X_t^i}. \tag{14}$$

By choosing the right-hand side, the minimal central bank is able to pin down the expected ratio of the stochastic discount factor and inflation. Inflation itself though is not determinate. There are an infinite number of inflation rates at different states of the world that satisfy this equation.[7]

If there is no uncertainty in the economy, the expectations operator disappears from equation (14). In that case, by choosing X_t^i, the central bank ensures a single Π_{t+1} at each date. Even then, there is no other condition to pin down P_0. If people expect higher prices in the future, the price level at 0 will simply jump up today, keeping inflation equal to $(1 + X_t^i)/(1 + R_t)$. The first condition for determinacy is not satisfied.[8]

2.1.2 Real payments on reserves

Imagine the central bank promises instead to remunerate reserve holders with a payment in real goods.[9] Governments have issued indexed bonds for a long time across the world, and so could central banks; this is what promising a real payment of goods amounts to. The nominal return on reserves in dollars would then be $1 + I_t = (1 + X_t^i)P_{t+1}$.

Plugging the above into equation (13) and rearranging delivers:

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - (1 + X_t^i)P_t \right) \right] = 0 \Rightarrow \; P_t = \frac{1 + R_t}{1 + X_t^i}.$$

[7] Nakajima and Polemarchakis (2005) provide a thorough discussion across different economic environments.

[8] This classic result is due to Sargent and Wallace (1975).

[9] This was studied by Hall and Reis (2016), building on earlier work by Hall (1997), which in turn formalized a proposal by Irving Fisher.

Since X_t^i is chosen by policy, and R_t is shaped by real forces, then the above equation delivers a determinate price level.

The intuition for how the price level is pinned down is the following. The real return on any investment is exogenously fixed by the stochastic discount factor. If the central bank promises a real payment on reserves, then arbitrage determines how many goods reserves are worth today. This is the economic force behind the Fisher equation: since real bonds and reserves both deliver the same payment tomorrow, they must be worth the same today. But, since reserves are denominated in dollars, not goods, then this pins down the price level today. No central bank does this, but this peg highlights the no-arbitrage forces behind inflation control.

The most effective rule is $1 + X_t^{i*} = (1 + \hat{R}_t)/\hat{P}_t^*$ leading to an effectiveness: $\varepsilon_t = r_t - \hat{r}_t + \hat{p}_t^* - p_t^*$. The better the estimates of the real interest rate, the more effective this policy will be.

2.2 Interest rate feedback rules

While picking interest rates on reserves, the central bank can choose a feedback rule to adjust the interest rate to inflation (or the price level).

2.2.1 The Taylor rule

The most famous of these rules is:

$$i_t = x_t^i + \phi \pi_t. \tag{15}$$

where $\phi > 1$, so the response to inflation is more than one-to-one.

The log-linearized version of the Fisher equation is:

$$i_t = r_t + \mathbb{E}_t(\pi_{t+1}). \tag{16}$$

Combining it with the Taylor rule to replace out the nominal interest rate delivers a difference equation for the deviations of inflation from target. Iterating forwards and imposing a terminal condition, such that $\lim_{T\to\infty} \phi^{-T}\, \mathbb{E}_t \left(\pi_{t+T} - \pi_{t+T}^*\right) = 0$, delivers a unique solution:[10]

$$\pi_t = \pi_t^* + \sum_{j=0}^{\infty} \phi^{-j-1}\, \mathbb{E}_t \left(r_{t+j} + \pi^*_{t+1+j} - \phi \pi^*_{t+j} - x^i_{t+j} \right). \tag{17}$$

Note that this equation holds for all $t \geq 0$. Since P_{-1} is given, the price level is determinate at all dates, including 0.

The most effective feedback rule sets the interest rate to respond to inflation as well as to the central bank's forecast of real interest rates and the inflation target: $x_t^{i*} = \hat{r}_t + \hat{\pi}^*_{t+1} - \phi \hat{\pi}^*_t$. Its effectiveness is:

$$\varepsilon_t = \varepsilon_{t-1} + \sum_{j=0}^{\infty} \phi^{-j-1}\, \mathbb{E}_t \left[r_{t+j} - \hat{r}_{t+j} + \pi^*_{t+1+j} - \hat{\pi}^*_{t+1+j} - \phi(\pi^*_{t+j} - \hat{\pi}^*_{t+j}) \right] \tag{18}$$

The right-hand side summarizes the public's expectations of the estimation mistakes made by the central bank on the state of the economy. Even if neither the central bank nor the public know what r_t or π_t^* are, and even if their estimates are poor, as long as these estimates coincide, the policy will be effective.

2.2.2 The virtues of transparency

The effectiveness of an interest rate feedback rule depends crucially on the public agreeing with the central bank's view of the current and expected future state of the economy. The literature has called Delphic forward guidance to central bank communications of what it thinks the future states of the economy will be.

The Federal Reserve started to release lightly edited transcripts of previous FOMC meetings in 1993. In 1999, it began issuing statements at the conclusion of every policy meeting, and in 2000 included in that statement a balance of risks. Orphanides (2019) summarizes some of this evolution in the context of anchoring inflation expectations when setting interest rates.

Across the world, central banks since the early 1990s started adopting inflation targeting frameworks. More than announcements of official targets for inflation, these consisted primarily of transparency and communication with the public about the central banks' objectives, plans, and actions (Bernanke and Mishkin, 1997). Empirical work in this area has shown repeatedly that communication works under inflation targeting by moving

[10] This equation also implicitly assumes that the infinite sum of expectations is finite. This is a weak requirement on policy: it must be sufficiently effective so that x_t^i does not stray away from the real interest rates and inflation exponentially over time.

financial markets, the economic force behind feedback rules (Blinder et al., 2008). Data for 112 countries from 1998 until 2019 shows an almost uniform increase in transparency measured by central banks releasing data, sharing their internal forecasts, explaining their framework and deliberations, and disclosing policy decisions and their rationale (Dincer et al., 2022). Designing a central bank today is as much defining goals and strategies as it is to set a framework for transparency and accountability (Reis, 2013), and the IMF even publishes a Central Bank Transparency Code with international standards (IMF, 2020).

2.2.3 Optimizing the rule

A large literature has discussed the general class of interest rate rules and estimated them using data across countries and time regimes.[11] Within the formulation of equation (15), a wide variety of measures and estimates of real activity have been included in the rule, since central banks often respond to recessions by cutting interest rates. These would fall under the x_t^i term in our notation. Since our analysis already allowed for (r_t, x_t^i) to be general stochastic processes—the only restriction was that they were exogenous with respect to inflation—we have already covered their determinacy. They only affect the effectiveness of the policy.

There are broader classes of feedback rules. First, most estimates of policy rules also show that interest rates are inertial. Central banks typically break a desired change in interest rates into 0.25% or 0.5% steps over successive policy meetings. We can represent this by having current interest rates responding to their own past value. Second, convinced by estimates that monetary policy only affects inflation with a lag, many central banks adjust interest rates in response to public forecasts of future inflation, obtained from surveys, financial prices or internal models of the central bank. We can capture this by adding the public's expectation of future inflation to the interest rate rule. Third, many central banks respond to core measures that try to smooth out the noisy real-time measures of inflation and capture its permanent trends. Following Muth (1960), we can model core inflation as a weighted average of past inflation, which is the optimal estimate if actual inflation follows a random walk contaminated with white noise measurement error. Fourth, studies of optimal monetary policy often suggest that the central bank should target the price level rather than inflation. These Wicksellian rules replace π_t with p_t in the policy rule.

[11] McCallum (1981) introduced these rules and first showed that they lead to determinacy. Taylor (1999), Clarida, Galí and Gertler (2000*a*) and Woodford (2003) are classic analyses.

Table 1: Determinacy conditions

Rule	*Condition*
Benchmark: $x^i_t + \phi\pi_t$	$\phi > 1$
Inertial: $x^i_t + \phi\pi_t + \chi i_{t-1}$	$\phi + \chi > 1$
Forecast targeting: $x^i_t + \phi\pi_t + \chi\,\mathbb{E}_t(\pi_{t+1})$	$\phi + \chi > 1$
Core inflation: $x^i_t + \phi(1-\chi)\sum_{j=0}^{\infty}\chi^j\pi_{t-j}$	$\phi > 1$
Wicksellian: $x^i_t + \phi p_t$	$\phi > 0$

The mathematics and economic logic of all these cases are similar to the ones in the analysis of the Taylor rule. Table 1 formalizes them and shows the determinacy conditions derived from the same steps as in the previous section. In all of them, the response of interest rates to inflation must be large enough, although the thresholds differ, as shown in the last column of the table. Also, all the formulas for effectiveness depend on the ability of the central bank to minimize the discrepancy between the public's and the central bank's forecasts of the state of the economy and the value of the inflation target. Depending on the relative variance and correlation between the state of the economy and the inflation target, some rules will be more effective than others.

2.2.4 The 1990s and 2000s experience

Monetary policy in the United States during the tenure of Alan Greenspan (1987-2006) closely conformed to what was prescribed by the rule of Taylor (1993). A simple version of equation (15) that includes the difference between the unemployment rate and a time-varying natural rate plus two lags of the Federal Funds rate has an R^2 of 0.97 on quarterly data during the Greenspan era, but much less during the time of his predecessor, Paul Volcker (Blinder and Reis, 2005). More broadly, feedback interest rate rules that satisfied the Taylor principle became the established way to conduct monetary policy across the world during the 1990s and 2000s (Leeson, Koenig and Kahn, 2013). The actions of the ECB, which started setting monetary policy during this period, can be well described in reference to a feedback rule for interest rates right up until 2013 (Hartmann and Smets, 2018).

During these close to twenty years, inflation was low and stable. Comparing every twenty-year period over eight centuries of UK inflation Reis (2023) finds that the period 1997-2016 had the best inflation outcomes. It had not been so before in all of the G-7 countries, and the most likely explanation was the new monetary regime (Cecchetti et al., 2007).

2.3 How does the Taylor principle work?

The arbitrage argument that pins down inflation was presented in an economy with the classical dichotomy. However, households' savings decisions on real and nominal investments condition their consumption of goods. In the presence of nominal rigidities, consumer demand affects how firms set prices. Does this fundamentally change how feedback rules work? Different rules put different lower bounds on ϕ for there to be determinacy. Since most central banks, from at least the 1990s, describe their policy as setting interest rates following a feedback rule, can we then not estimate ϕ and test this mechanism? What is the intuition for why moving from a peg to a feedback rule makes such a difference? We now answer these three questions.

2.3.1 Nominal rigidities

Defining the output gap as $\tilde{y}_t \equiv y_t - y_t^n$, there are three relevant equations:

$$\pi_t = \beta\,\mathbb{E}_t(\pi_{t+1}) + \kappa\alpha\tilde{y}_t + z_t \tag{19}$$

$$\tilde{y}_t = \mathbb{E}_t(\tilde{y}_{t+1}) - (i_t - \mathbb{E}_t(\pi_{t+1}) - r_t^n) \tag{20}$$

$$i_t = x_t^i + \phi\pi_t + \phi_y\tilde{y}_t. \tag{21}$$

The first is the Phillips curve in equation (6). The second rewrites equation (5) using the definition of the output gap, combining it with the Fisher equation and setting $\theta = 1$ for simplicity. Note that r_t^n is the equilibrium interest rate in the absence of nominal rigidities. This equation is often referred to as the IS curve. The third equation makes explicit that the interest rate feedback rule in (15) includes the output gap.

The steps to solve for inflation are the same, but now one must solve for the output gap at the same time. Eliminating the interest rate from the system we have two equations:

$$\begin{pmatrix} \tilde{y}_t \\ \pi_t \end{pmatrix} = \Phi \, \mathbb{E}_t \begin{pmatrix} \tilde{y}_{t+1} \\ \pi_{t+1} \end{pmatrix} + \Omega \begin{pmatrix} r_t^n - x_t^i \\ z_t \end{pmatrix}, \tag{22}$$

$$\text{where } \Phi \equiv \Omega \begin{pmatrix} 1 & 1-\beta\phi \\ \kappa\alpha & \kappa\alpha + \beta(1+\phi_y) \end{pmatrix} \quad \text{and} \quad \Omega \equiv \frac{1}{1+\phi_y+\kappa\alpha\phi}.$$

A system of linear difference equations has a unique non-explosive solution if the number of eigenvalues of the matrix outside the unit circle is equal to the number of non-predetermined variables. In this case, both output and inflation can in principle jump, so both eigenvalues have to have modulus larger than 1. Standard linear algebra shows that this is the case if the following condition holds:

$$\phi > 1 - \frac{\phi_y(1-\beta)}{\kappa\alpha}. \tag{23}$$

This is a generalized version of the Taylor principle condition for determinacy above. The coefficient on the output gap relaxes the condition because output covaries with inflation in the long run in this model, so that by responding to output, the central bank is indirectly further responding to inflation.[12] The same intuition carries through, together with the reliance on a terminal condition.

The characterization of the effectiveness of the interest rate feedback rule as a monetary policy approach is also similar. The control errors are again a discounted sum of the public's perceived deviations between natural rates of interest, inflation targets, and, now, markups over time, but with different weights.

The prevalence of a real indeterminacy, in addition to the nominal one, with presence of nominal rigidities brings a further economic force at play. Besides the no-arbitrage channel that is specific to this approach there is an aggregate demand channel as well. Changes in the return of financial assets affect households' desire to save, while nominal rigidities make output demand determined. Therefore changes in the interest rate now also affect inflation through changes in consumption.

In short, nominal rigidities and considering the two-way interaction between inflation and output does not change significantly how feedback rules for interest rates work in this

[12] Meyer-Gohde and Tzaawa-Krenzler (2023) show that in models of the Phillips curve, like sticky information, where fully anticipated long-run monetary policy has no effect on output, the condition reverts back to $\phi > 1$.

economy.[13] It adds an important complementary demand channel, which interacts with the no-arbitrage forces to pin down the price level.

2.3.2 Testing the Taylor principle

If one goes by the speeches, reports, and statements of central banks, one would think that they all follow feedback rules and subscribe to the Taylor principle. But central banks say many other things as well, and it turns that it is very hard to empirically verify the condition for determinacy.

Going back to the solution for inflation in equations (17) and (18), imagine that the central bank manages to be fully effective, so $\varepsilon_t = 0$ and $\pi_t = \pi_t^*$ at all dates. In that case, the nominal interest rate will be $i_t = x_t^i + \phi\pi_t^* = r_t + \pi_{t+1}^*$. Since everything is exogenous on the right-hand side, this rule is observationally equivalent to a peg. Even if the econometrician had data allowing her to separate the state of the economy r_t from desired inflation π_{t+1}^*, she could not estimate ϕ.

Imagine instead $r_t = \pi_{t+1}^* = 0$, so that there are no shocks to the economy or to the policy goal, but only to monetary policy (mistakes) that follow the stationary process $x_t^i = \rho x_{t-1}^i + \epsilon_t$, where ϵ_t is iid mean zero. Then, the solution in equation (17) reduces to $\pi_t = -x_t^i/(\phi - \rho)$ so inflation is also autoregressive of order 1. Now, solving for the interest rate: $i_t = x_t^i + \phi\pi_t = -(\phi - \rho)\pi_t + \phi\pi_t = \rho\pi_t$. Therefore, a regression of nominal interest rates on inflation would recover the parameter ρ. Since $\rho < 1$ this estimate would mislead the econometrician to think the Taylor principle is violated (Lubik and Schorfheide, 2004, Cochrane, 2011).

The more general principle is that since shocks to the feedback rule affect inflation, regressions of nominal interest rates on inflation inevitably give biased estimates of the feedback coefficient. Only when $x_t^i = 0$ or the econometrician can perfectly observe x_t^i to include in the regression is the estimate from that regression ϕ.[14]

2.3.3 The elusive terminal condition

If it is so hard to test for the Taylor principle, how can its presence ensure determinacy? Imagine that inflation is higher at date t by one log unit relative to the solution above.

[13] Allowing for incomplete markets in the presence of nominal rigidities, as we eventually do in section 5, also does not change the intuition underlying feedback rules. Bilbiie (2018) provides the corresponding version of the Taylor principle in an heterogeneous agent New Keynesian framework.

[14] Carvalho, Nechio and Tristao (2021) argue that as long as the output gap is used to proxy for x_t^i, what is left that drives policy has a small enough variance that the bias will be small.

Then, with the Taylor rule the central bank will raise the nominal interest rate by ϕ leading to an increase in expected inflation between t and $t+1$ of ϕ (the logic is the same for the other rules). But this in turn leads the central bank to raise i_{t+1} by ϕ^2, which raises expected inflation between $t+1$ and $t+2$ by that amount. The process continues so inflation keeps on rising exponentially and the feedback rule imposes inflation in T periods to be larger by ϕ^T.

The inflation target level is the unique possible solution, because the following terminal condition ruled out these deviations:

$$\lim_{T\to\infty} \phi^{-T}\, \mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T}\right) = 0. \tag{24}$$

Equivalently, the random variable $\mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T}\right)$ belongs to $O(\ln(\phi))$. That is, if expected inflation deviates from target, those deviations cannot grow faster than at the rate $\ln(\phi)$. The larger is ϕ, the weaker is this condition. But where did that condition come from in the first place?

The terminal condition is not an optimality condition, the way that transversality conditions are. Those apply to the real value of savings, whereas the condition needed here is on a purely nominal variable, the price level. Additionally, optimal behavior imposes no money illusion in the Euler equation or in the transversality condition. Furthermore, there is no sense in which the economy blows up if this condition does not hold. In the classical economy, the unit of account may be exploding, but agents with no money illusion would be indifferent as real outcomes and variables continue to be finite. With nominal rigidities, real outcomes would explode, but assuming that prices would remain sticky as inflation shoots to infinity is absurd.

The most common justification for the terminal condition is that the equilibria that violate it are not plausible. The feedback rule ensures that any of these equilibria associated with indeterminacy leads to explosive paths for inflation. Perhaps people would never believe them. More formally, if people's expectations of inflation deviations from target in the future are constrained to stay locally bounded, then $\mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T}\right)$ is $O(0)$ and the Taylor principle implies the terminal condition. Among the set of bounded equilibria, inflation is determined.

A related argument notes that since the derivations above relied on log-linearization of the Fisher equation, inflation should be bounded for the error to be small in this local approximation. Restricting attention to bounded equilibrium is coherent with how the model is being solved. But there is no strong argument for why.[15]

2.4 Escape clauses as anchors

Terminal conditions can be given by escape clauses. The idea is that the central bank commits to a feedback rule only while inflation does not go on an explosive path. If inflation exceeds a pre-announced threshold, the central bank would switch to a different policy approach. Realistically, if inflation was rising without bound, no central bank would stick to following blindly a Taylor rule that tells it to raise interest rates more and more, even as it sees inflation rising faster and faster.

2.4.1 On-equilibrium policy switches

If the approach dictated by the escape clause pins down the price level at the date of the switch, then it provides the terminal condition for the feedback rule.

Formally, the central bank follows the feedback rule only while inflation is within some interval $[\pi^L, \pi^H]$. If, at some date T, inflation π_T falls outside this interval, then it switches to a different policy at $T+1$. Take as given that this other policy is able to determine uniquely π_{T+1} as close as possible to the target π^*_{T+1}. It could, for instance, set a real payment on reserves as we already saw, or involve a different approach like fixing the supply of banknotes. This paper will discuss many approaches to pin down π_{T+1} further on.[16]

Going back to the solution for inflation with a Taylor rule, by iterating the Fisher equation up until a finite date T, we reach:

$$\pi_t = \pi_t^* + \sum_{j=0}^{T-t} \phi^{-j-1}\,\mathbb{E}_t\left[r_{t+j} + \pi^*_{t+1+j} - \phi\pi^*_{t+j} - x_{t+j}\right] + (1+\phi)^{-T+t}\,\mathbb{E}_t\left(\pi_{T+1} - \pi^*_{T+1}\right). \tag{25}$$

If the last term on the right-hand side is uniquely pinned down by the switch in regime, then inflation on the left-hand side is uniquely pinned down as well. If the switch leads to an inflation close to target, then the last term will be close to zero. Therefore, the effectiveness is still approximately given by the formula for ε_t that we derived earlier for the Taylor rule.

[15] Cochrane (2011) makes a scathing critique of these arguments.

[16] The classic analysis is Obstfeld and Rogoff (1983), and see also Taylor (1996) and Christiano and Rostagno (2001).

Of course, if either the width of the interval $[\pi^L, \pi^H]$ goes to zero, or the errors ε_t are large enough, then T would be close to 0. The economy would switch policy regime right away. The case for feedback rules is that it achieves lower ε_t than the alternatives and that T is expected to be large, so that leaving the interval is infrequent.

Arguably, this is the case for the ECB, which has a monetary pillar, understood as a commitment to switch to a monetary approach to pin down inflation if inflation starts exploding. It also serves as an alert to central banks that have successfully used feedback rules to control inflation for decades and think this is enough. Even though T may be large, it is finite, and having a monetary anchor as an escape clause behind the interest rate rule may be crucial. Many central banks have such monetary anchors, often in the form of gold reserves, or holdings of foreign currency, even if they are rarely used.

2.4.2 Off-equilibrium threats

Regime switches can be used differently, not as terminal conditions, but as off-equilibrium threats that ensure that the regime switch never happens.[17]

Say policy is still committed to a Taylor rule while inflation stays in a bounded interval $\left[\pi^L, \pi^H\right]$. If at date T inflation π_T is outside of it, there is a switch in policy at $T+1$. The new policy is able to uniquely pin down inflation π_{T+1} as before.

The difference is that now this exit will never happen because it is inconsistent with equilibrium. The new policy is designed to pin down inflation to some level well inside the interval, and in particular to a level such that $\pi_{T+1} < \pi^H - r_t$.[18] The Fisher equation (16) at date T together with the regime switch pins down $i_T = \pi_{T+1} + r_t < \pi^H$. At the same time, the Taylor rule at T implies that since π_T was larger than π^H, and given that the Taylor rule coefficient is larger than one, $i_T > \pi^H$. This is a contradiction.

The only way to avoid the contradiction is for inflation to never leave the bounded interval. If the width of the interval is large enough such that the size of the exogenous shocks would never send the economy outside the interval, then the explosions that lead to indeterminacy with a Taylor rule are ruled out, but needed fluctuations due to changes in the inflation target are not. As the feedback rule implies that inflation explodes at rate ϕ^{-1}, then one of the bounds will be reached for sure in finite time for any inflation path that does not satisfy the elusive terminal condition. Thus the condition holds.

[17]Much of this work builds on Bassetto (2005), and includes Atkeson, Chari and Kehoe (2010)'s sophisticated equilibria, Christiano and Takahashi (2018)'s strategy equilibria and Loisel (2021)'s implementability criteria.

[18]This may require nominal rigidities to make the policy consistent by having deviations from it be costly.

Just like in the previous case, the central bank is making the promise that it will not stick to the Taylor rule if inflation enters one of the explosive paths that violate the terminal condition. But now, the escape clause is inconsistent with equilibrium, and so it is assumed that rational agents would never expect it to be used. This requires a great deal of commitment by the central bank.

2.4.3 The 1970s experience and the monetary pillar

Inflation was high during the 1970s across most advanced economies. One explanation for why this happened is that central banks did not satisfy the Taylor principle in setting interest rates (Clarida, Gali and Gertler, 2000*b*, Coibion and Gorodnichenko, 2011). A complementary explanation is that policymakers at the time settled for a higher inflation target π^* (Meltzer, 2010), mis-estimated the state of the economy r_t (Orphanides, 2003), and neglected the measurement and management of private-sector expectations (Reis, 2021). Each of these, and all combined, could have contributed to inflation exploding as a result of either indeterminacy or the lack of a terminal condition anchoring expectations. The conquest of US inflation by Paul Volcker after 1979 came with a brief switch to monetarism (explained in a later section), precisely what an escape clause would dictate.

An exception to the dismal inflation performance during the 1970s was West Germany. The Bundesbank, while also setting interest rates that satisfied the Taylor principle during this time, had targets for monetary growth that made explicit the potential switch to a money strategy if inflation ever got too far from target (Clarida and Gertler, 1997). This experience had an important impact on the design of the ECB twenty years later as following a two-pillar strategy. As described in Rostagno et al. (2021), from the 1998 initial strategy to the 2003 review, the ECB emphasized the flexibility of potentially switching to monetarism as a pillar that would stabilize expectations.

3 Unconventional interest rate policies

Starting in 1999, the Bank of Japan found itself unable to use feedback interest rate rules to control inflation. The same happened to the Federal Reserve, the ECB, and other major central banks at some time after the great financial crisis. The reason was that the rule prescribed setting a very low policy rate, well below zero. Central banks throughout the 2010s continued to rely on the forces of no arbitrage and to use the interest rate as their policy tool, but now did it in different ways, which were labelled unconventional in spite of their persistent use (Bernanke, 2020). As much or more than before, they relied on expectations of private agents into the future, often far away. This section discusses these alternative interest rate policies.

In preparation, we start by asking whether our previous conclusions were restricted to small fluctuations in inflation since they relied on log-linearizations around steady states. To simplify, consider the case where there is no uncertainty, so the stochastic discount factor is a constant β and the inflation target Π_t^* is deterministic. Therefore, there is possibly indeterminacy only with respect to the initial price level. The nonlinear Taylor rule is $1 + I_t = \Pi_t^{\phi} X_t^{i*}$, while the Euler equation for nominal bonds (7), reads: $\beta(1 + I_t) = \Pi_{t+1}$. Combining the two, and assuming the most effective rule $X_t^{i*} = \Pi_{t+1}^* / (\beta \Pi_t^{*\phi})$, gives:

$$\frac{\Pi_{t+1}}{\beta} = \Pi_t^{\phi} \left(\frac{\Pi_{t+1}^*}{\beta \Pi_t^{*\phi}} \right). \tag{26}$$

This simplifies to the nonlinear difference equation: $\Pi_{t+1}/\Pi_{t+1}^* = (\Pi_t/\Pi_t^*)^{\phi}$.

Taking logs gives precisely the same dynamics as in the log-linearized case. If inflation starts on target, it stays there forever as long as $\phi > 1$. If it deviates upwards or downwards, then this leads to inflation exploding to plus or minus infinity at the rate $\ln(\phi)$. Inflation control depends again on a terminal condition that rules out these equilibria.

3.1 Banknotes and the zero lower bound

In the case of explosive deviations of inflation downwards, at some point inflation will go below the real interest rate so the nominal interest rate must be negative as well. Yet, central banks are committed to exchanging their reserves for banknotes one for one, and banknotes have a gross nominal return ξ that is below but close to 1, since they pay no interest but have storage costs and risk of theft. Banks would want to substitute all of their reserves for banknotes if interest rates went below ξ. Banknotes impose an effective lower bound (ELB) on the payment of interest on reserves.

Any monetary policy rule that remunerates reserves in a way that is inconsistent with the ELB is not admissible. Consider first a payment on reserves rule, where determinacy was not an issue, so only effectiveness was at stake. The rule has to be modified to: $1 + I_t = \max\{(1 + X_t^i)P_{t+1}, \xi\}$. When the ELB does not bind, say at date T, one gets the same equations as before, pinning down the price level on target at P_T^*. When it does bind, then the Fisher equation implies that $P_{t+1} = \beta \xi P_t$. Because P_T was pinned down, so are all the prices before then, and the price level is still determinate. However, now, $P_t = (\beta\xi)^{t-T} P_T^* \neq P_t^*$. During the ELB periods, the economy is in a deflation, no matter what the inflation target is, and the central bank can do nothing about it.

Turning to the more interesting case of the Taylor rule, the problem gets more serious (Benhabib, Schmitt-Grohe and Uribe, 2001). From the Fisher equation, the ELB constraint $1 + I_t \geq \xi$, implies that $P_{t+1}/P_t \geq \beta\xi$. Combining with the Taylor rule, inflation becomes:

$$\frac{\Pi_{t+1}}{\beta} = \max \left\{ \frac{\Pi_{t+1}^*}{\beta\xi} \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\phi}, 1 \right\}. \tag{27}$$

As soon as inflation is equal to $\beta\xi$, it stays there forever. This is a global steady state equilibrium of the difference equation: a deflation trap. Moreover, if P_0 is below target, inflation will fall but instead of exploding, it will now converge to the deflation trap. This is true for any initial P_0, so the price level is again indeterminate: any initial inflation between Π_0^* and $\beta\xi$ is consistent with an equilibrium.[19]

Setting aside indeterminacy, the presence of two steady states in this system, one where $\Pi_t = \Pi_t^*$ and another where $\Pi_t = \beta\xi$ implies that in a system with shocks to either the state of the economy, the inflation target, or policy mistakes, there will be two stochastic solutions fluctuating around these steady states. Then, if there is a sunspot that triggers a change between them, equilibrium inflation will alternate between being close to target or being close to the deflation trap (Mertens and Ravn, 2014, Aruoba, Cuba-Borda and Schorfheide, 2017). Depending on the exogenous distribution of this sunspot, the effectiveness of policy can be arbitrarily poor.

In principle, one can eliminate the deflation equilibrium with an escape clause. However, with an inflation target of 2%, deflation is never too far, and escapes would be too frequent, making the feedback rule not too useful. Alternatively, one could eliminate or relax the ELB constraint by lowering ξ, perhaps all the way to zero. Some suggestions in the literature on how to lower ξ are to eliminate banknotes, charge a tax on them, or default on the commitment to exchange currency and reserves one-for-one (Goodfriend, 2016, Rogoff, 2017, Agarwal and Kimball, 2019).

[19] Christiano, Eichenbaum and Johannsen (2018) show that an e-stability restriction on the set of equilibria delivers uniqueness.

3.2 Nominal rigidities and forward guidance

Price rigidities interacting with the effective lower bound have occupied a large strand of literature (Eggertsson and Woodford, 2003). While this is not the place to survey it, we focus on how it affects the dynamics of inflation. Again, we consider an economy that is at the effective lower bound from period 0 to T. As we saw in the previous section, an interest rate feedback rule and a terminal condition given by an escape clause will pin down inflation from date T onwards. As before, nominal interest rates paths from date T onwards do not pin down inflation by themselves, and different paths of inflation from T onwards will lead to different paths of inflation before date T (Werning, 2011, Cochrane, 2017). Also, once again, there is a permanent-deflation equilibrium that arises from a global analysis, since nominal rigidities do not bind at a steady state. However, now small changes in how the nominal rigidities are modeled, including whether prices are sticky as in Calvo (1983) or as in Rotemberg (1982), or in how the sunspots that coordinate the equilibria are introduced, or even in what numerical methods are used to characterize the global solution, seem to matter significantly for the properties of the equilibrium.[20]

The path of inflation while the economy is at the ELB is different though, and points to an unconventional interest rate policy. With the classical dichotomy, we showed that inflation before date T is negative and given by $P_t = (\beta\xi)^{t-T} P_T$. This is still true with nominal rigidities. Combining equations (19) and (20) replace out output and assuming away shocks for simplicity ($y_t^n = z_t = 0$) gives a second-order difference equation for inflation:

$$\pi_t = (1 + \beta + \kappa\alpha)\pi_{t+1} - \beta\pi_{t+2} - \kappa\alpha(i_t - r_t^n). \tag{28}$$

Now, during the period when the ELB binds $i_t = \ln(\xi)$. Since π_T and π_{T+1} are determined, there are two terminal conditions for this equation to give the whole path of inflation from 0 to $T-1$. Just as in section 3.1, the central bank has no power to affect this path for inflation, which may be very far from the target inflation rate. During this path, deflation comes with output below its natural level (a recession).

One property of this system is that the larger is T, the lower is inflation and output at date 0. In the limit, a temporary interest rate peg that lasts forever has an unboundedly large effect on inflation and output today. Related, imagine that for a fixed number of periods $T^Z < T$, we have $r_t^n = \underline{r} < \ln\xi$, making it impossible to achieve a $\pi_t^* = 0$ target, but that between T^Z and T the central bank chooses to keep the nominal interest

[20] See Fernández-Villaverde et al. (2015), Boneva, Braun and Waki (2016), and Christiano, Eichenbaum and Johannsen (2018) among others.

rate at $\ln \xi$ even though $r_t^n = 0$. The period between T^Z and T is a period of strict *forward guidance*: the central bank is keeping the nominal interest rate at the ELB even though it was not constrained by the state of the economy to do so. To distinguish the unconventional announcement of a path for the policy rate, from the conventional communication of the state of the economy that we already discussed, this is sometimes called instead Odyssean forward guidance.[21]

Then, the second-order difference equation above has a startling property: the larger is T, keeping T^Z fixed (that is the larger is the period of forward guidance), the higher are inflation and output at date 0. In fact, if forward guidance is long enough, output may even go above the natural level at date 0. The combination of the ELB with the Calvo Phillips curve makes forward guidance in the distant future a powerful tool to control inflation in the present.

This result has been called the forward guidance puzzle since it is easily contradicted by empirical estimates of the effects of forward guidance.[22] At the same time, the literature has found that limits to rationality, incomplete insurance markets that change the IS relation in equation (20), or different models of price rigidity like sticky information that change the Phillips curve in equation (19) can make the puzzle go away.[23]

3.3 Quantitative easing and going long

Over the last twenty years, central banks went long in the sense that the focus of monetary policy became long-term interest rates. The Bank of Japan went the furthest by announcing a desired target for the 10-year interest rate, standing ready to buy and sell government bonds of this maturity to hit the target. The Fed and the ECB engaged in quantitative easing whereby their purchased long-term bonds with reserves with the goal of lowering long-term interest rates.

In theory, if the central bank issued bonds of a fixed maturity that were later paid off with reserves, it could choose how to remunerate these bonds just as it does with

[21] Disentangling Delphic from Odyssean forward guidance is empirically challenging even with high-frequency data (Gürkaynak, Sack and Swanson, 2005, Campbell et al., 2017, Andrade and Ferroni, 2021).

[22] The puzzle was identified in Del Negro, Giannoni and Patterson (2012) and Carlstrom, Fuerst and Paustian (2015).

[23] Angeletos and Lian (2018), Gabaix (2019), and García-Schmidt and Woodford (2019) study deviations from perfect-foresight rationality in this context, Del Negro, Giannoni and Patterson (2012) and McKay, Nakamura and Steinsson (2016) explore incomplete insurance against income risks by households, and Carlstrom, Fuerst and Paustian (2015), Kiley (2016), Eggertson and Garga (2019) explore sticky information price rigidities.

reserves. If the central bank issues a j period bond and pays I_t^j interest rate on it, then the Euler equation that applies to this new form of investment is:

$$\mathbb{E}_t\left[\frac{M_{t,t+j}(1+I_t^j)}{\Pi_{t+1}\Pi_{t+2}...\Pi_{t+j}}\right]=1. \tag{29}$$

The stochastic discount factor between two non-successive dates is: $M_{t,t+j}=M_{t+1}M_{t+2}...M_{t+j}$. By choosing a feedback rule for I_t^j in much the same way as it did for one-period reserves, the central bank can control the price level. The condition for determinacy still requires ϕ to be larger than some threshold, but the threshold is now equal to the sensitivity of long rates to short rates. The effectiveness of this policy involves similar terms but with different weights (McGough, Rudebusch and Williams, 2005, Reis, 2019*b*).

Alternatively, the central bank may choose short-term and long-term interest rates simultaneously. In this case, the Euler condition provides an extra set of equations, one for each date t. Increasing the number of equations without increasing the number of unknowns gives hope that perhaps inflation is now determinate (Adão, Correia and Teles, 2014, Magill and Quinzii, 2014).

To see this at play, consider the simple case in which there is only uncertainty about M_{t+1}, which follows a two-state stationary Markov chain with values M_H and M_L and transition matrix with non-negative probabilities satisfying $f_{HH}+f_{HL}=1$ and $f_{LH}+f_{LL}=1$. Controlling inflation boils down to determining the two values of inflation, Π_H and Π_L, uniquely. The Euler equations with respect to the one-period reserves and the two-period bonds can be written at state s as:

$$\begin{aligned}(1+I_s^1)\left(f_{sH}\frac{M_H}{\Pi_H}+f_{sL}\frac{M_L}{\Pi_L}\right)&=1, \\ (1+I_s^2)\left(f_{sH}\frac{M_H}{\Pi_H(1+I_H^1)}+f_{sL}\frac{M_L}{\Pi_L(1+I_L^1)}\right)&=1.\end{aligned} \tag{30}$$

These are two equations in two unknowns. Standard linear algebra shows that as long as $I_H^1\neq I_L^1$, then there is a unique solution for inflation. The key condition for determinacy is now that the central bank does not set the interest on reserves to be the same across states of the world. Similar steps show that if the central bank announces both its current interest rates on reserves, as well as its expected value for tomorrow, this again provides two equations with which to solve for inflation across states.

Note that this approach does not pin down P_0. Only the stochastic degree of indeterminacy disappears. Intuitively, both the mean of inflation as well as how it covaries with the stochastic discount factor across two successive periods is now pinned down by arbitrage. Thus, the indeterminacy of inflation across states of the world can be solved as long as the nominal interest rate varies with those states of the world. However, while these interest rates are varying over states, over time they are still pegged in the sense of the interest rate peg. Thus, the problem of controlling P_0 remains.

While quantitative easing over the 2010s had an immediate effect on long-term interest rates, the literature has struggled to find a sizeable and persistent impact on inflation (Krishnamurthy and Vissing-Jorgensen, 2013, Fabo et al., 2021). Historically, the source of going long has instead often been the Treasury rather than the central bank. Especially in the aftermath of wars, when long-term government debt is high, fiscal policy imposes low long-term interest rates. Whether it is the Treasury or the central bank that uses its power to steer interest rates, the economics underlying the effect on inflation is the same.

3.4 The cost of credit

Long-term interest rates may be especially important once one considers nominal rigidities. If financial institution arbitrage between holding long-term bonds and making loans to firms, the central bank can affect the cost of credit, the marginal cost of production directly, and so the prices set by firms. Another unconventional tool used in the 2010s were credit policies, whereby the Bank of England (through the Funding to Lending scheme) and the European Central Bank (through the Targeted Long-term Repurchase Operations) lent funds to banks at favorable rates under the condition that these funds would then be used to provide loans to firms.

To analyze the effect of these policies requires a slight modification of the model. The cost of inputs Q_t, so far depended solely on the marginal disutility of labor and, through it, on the level of output. Conceivably, credit is an input in production, and if bank credit rates rise, then so will Q_t, raising marginal costs. With flexible prices, this real cost of credit would still be determined with other real variables independently of inflation. It takes a nominal rigidity, like loans being set in sticky nominal amounts, or their rates in sticky nominal units, for this to lead to another transmission channel of monetary policy over inflation. Through credit policy, the central bank can then affect the real costs of credit, marginal costs of production, and through the Phillips curve, the optimal price set by firms (Christiano, Trabandt and Walentin, 2010, Fiore and Tristani, 2013).

Beyond firm credit, similar mechanisms could operate through household credit, especially on mortgages that have features set in sticky nominal terms, affecting demand for goods as opposed to supply (Greenwald, 2018, Berger et al., 2021). A third channel through which lending rates can affect credit is if they affect the net worth of borrowers and tighten borrowing constraints (Bernanke, Gertler and Gilchrist, 1999).

While there is strong evidence for a credit channel of monetary policy (Ciccarelli, Maddaloni and Peydró, 2015, Gertler and Karadi, 2015), using credit supply or credit rates as the main strategy to control inflation is rarely used today. When tried in the United Kingdom in the 1950s in the context of the Radcliffe report, it failed (Capie, 2010). Central banks have an influence on lending conditions, but are very far from controlling them. There are large financial shocks in the lending markets that would translate into large fluctuations in inflation.

3.5 Non-rational expectations

These unconventional policies require affecting expectations in financial markets. Moreover, since the Taylor rule requires that people do not start expecting that inflation in an arbitrary far-away future will grow (or fall) at an explosive rate, it relies heavily on rational expectations into the infinite future. Relaxing rational expectations on far-away events becomes important.

The literature on non-rational expectations is too rich to cover here and has already been reviewed by Woodford (2013). Instead, we just describe three approaches that have been used to study the control of inflation. The first are learning models that assume that expectations are formed by agents that behave like statisticians using past data to form their beliefs. Learning gives a mapping from past outcomes to current expectations. In turn, inflation with an interest rate rule solves $\phi\pi_t = r_t + \mathbb{E}_t(\pi_{t+1}) - x_t^i$, which maps expectations into outcomes (this is sometimes called a temporary equilibrium). Combining the two gives the learning equilibrium.

The most popular is least-squares learning, where agents use least-squares regressions on past outcomes to form their beliefs. Taking the limit as the sample on which these regressions are run goes to infinity delivers what is known as the learnable equilibrium. The literature focuses on the e-stability principle, that establishes that learning converges to the non-explosive rational expectations equilibrium if certain stability conditions hold. For either learnability or e-stability, in our simple model with constant r and x^i, the non-explosive rational expectations equilibrium inflation is $\pi^{RE} = (r - x^i)/(\phi - 1)$, and one can show that $\phi > 1$ makes this learnable and e-stable (Evans and Honkapohja, 2001, Bullard and Mitra, 2002, McCallum, 2003).

Another popular class of non-rational expectations models are models of e-duction. Their central idea is that agents go through a mental process whereby they iterate on what expectations to have, and what their implications are for equilibrium inflation, until the two converge. This convergence need not happen at the fixed point of rational expectations, nor does it have to happen over time, like with learning, but rather occurs in agents' minds. For instance, with reflective expectations, at each stage of inference, agents update their expectations to close the gap to the expectations that are model consistent. In that case, it turns out that in the limit, as the rounds of reflection go to infinity, only the non-explosive rational expectations equilibrium is selected.[24]

Third, there are models of discounting the future through limited foresight (Gabaix, 2020) or the past through imperfect memory (Angeletos and Lian, 2023). Both imply that, either looking forward or backwards, current inflation depends less on far-away expectations. Because of that, both can deliver determinacy of inflation without escape clauses and with conditions on ϕ that are less strict than the Taylor principle. Sometimes, the limits to rationality or information are enough to select a single one of the multiple equilibria that arise even with an interest rate peg.

More generally, once one entertains non-rational expectations, then measuring expectations becomes important as an independent source of data and shocks. As much, or more, than measures of the output gap or natural rates of interest, these data on inflation expectations become part of the "state of the economy" x_t^i that an effective policy rule should include to keep inflation near its target (Reis, 2022).

4 The monetarist approach: currency, seignorage, and pegs

A log-linearized version of the Fisher equation (13) together with the demand for currency in equation (8), assuming log utility, gives:

$$h_t - p_t = c_t - \eta(r_t + \mathbb{E}_t \pi_{t+1}) + u_t. \tag{31}$$

where $\eta = (1 + I)^{-1}$. The income elasticity of the demand for banknotes equal to one is consistent with the empirical fact that the inverse of velocity, H_t/C_tP_t, has not displayed

[24] See García-Schmidt and Woodford (2019), building on the calculation equilibrium of Evans and Ramey (1992) for the reflective case, and Farhi and Werning (2019) for k-level thinking.

a strong trend over recent decades. [25]

The u_t represents a shock to the demand for central bank currency. There is a disconnect between the banknotes the central bank prints and the money that people find useful given the existence of close substitutes to currency produced by the private market. This is captured by u_t, with estimates that are large and volatile as a result of changes in the availability of ATMs, in the social norms of what shopkeepers will accept as payment, or in the prevalence of crime that drives the demand for the anonymity of banknotes.

The economic force that drives the price level behind this equation works as follows. All else equal, a higher price level today lowers real currency balances supplied by the central bank. At the same time, it lowers expected inflation between the present and the next period, which lowers the nominal interest rate and raises the demand for banknotes. With lower supply and higher demand for banknotes, the price level must fall. This re-equilibrates the market by both increasing the supply, and by lowering demand through a higher nominal interest rate.

The logic is soothingly familiar because it reintroduces Marshallian partial-equilibrium supply and demand to think about the price level in terms of the service provided by banknotes. At the same time, it can be misleading because p_t is not the price of the banknotes. Changes in p_t bring the market to equilibrium by affecting both the actual cost of currency i_t *and* also by directly changing the quantity of real currency that is held.

4.1 Money growth rules

With the stock of banknotes h_t as the new policy tool, the policy strategy is to set it following a rule that delivers inflation through equilibrium in the money market.

4.1.1 Constant money growth

The classical monetarist rule proposes that the supply of currency grows at a constant rate over time: $h_t = \bar{x}^h t$, where $\bar{x}^h$ is a constant. Replacing into equation (31) gives a difference

[25] There is a long empirical literature devoted to estimating this function. Benati et al. (2021) use data for many countries to obtain an interest rate elasticity between 0.3 and 0.6, while Alvarez and Lippi (2014) report an interest rate elasticity between 0.25 and 0.46. Ireland (2009) and Ball (2001) argue that a demand system relating log real currency balances to the level of interest rates fits the data better, but the former estimates a semi-elasticity of demand of 1.8-1.9, while the latter estimates it to be only 0.05 once one allows the income elasticity to be below 1 (and estimated to be 0.5).

equation for the price level:

$$(1+\eta)(p_t - \bar{x}^h t) = \eta\, \mathbb{E}_t(p_{t+1}) - \eta \bar{x}^h t + \eta r_t - c_t - u_t. \tag{32}$$

As before, we can iterate this forward as long as $\eta > 0$.[26] But now, there is a concrete terminal condition: the transversality condition in equation (11). When the only asset in non-zero net supply is currency, it is a terminal condition that follows from optimal behavior:

$$\lim_{T\to\infty} M_{t,T}\left(\frac{H_T}{P_T}\right) \approx \lim_{T\to\infty} \beta^T \left(h_T - p_T\right) = 0. \tag{33}$$

The price level is thus determinate and given by:

$$p_t = \bar{x}^h t + \eta \bar{x}^h + \frac{1}{1+\eta}\sum_{j=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^j \mathbb{E}_t[\eta r_{t+j} - c_{t+j} - u_{t+j}]. \tag{34}$$

Without currency shocks, in a long-run balance growth path where consumption grows at a constant rate, inflation is equal to the money growth rate $\bar{x}^h$ minus the growth rate of consumption. Thus, choosing $\bar{x}^h$ to be the long-run inflation target of the central bank plus the real growth rate of the economy provides an effective way to achieve the target.

4.1.2 The link to interest rate rules

We can rewrite the equilibrium in the currency market in equation (31) as:

$$i_t = \frac{p_t}{\eta} + \frac{c_t + u_t - h_t}{\eta}. \tag{35}$$

This is mathematically equivalent to a Wicksellian interest rate feedback rule. Since $1/\eta > 0$, it satisfies the determinacy condition. But while in section 2 this was a policy rule, here it emerges as an equilibrium condition.

The link to interest rates under a monetarist approach arises because the nominal interest rate i_t adjusts endogenously so that the market for currency clears. Canzoneri, Henderson and Rogoff (1983) blur this distinction by specifying a feedback rule for currency that depends on the nominal interest rate: $h_t = x_t^h + \phi i_t$. In this case, the central

[26]Different micro-foundations for money imply different forms of the money demand function, and as such potentially slightly different conditions for determinacy of the price level. Still, the basic result and economic intuition remains, see Carlstrom and Fuerst (2003).

bank can limit the volatility of the nominal interest rate. In fact, it can even peg it to follow a pre-determined path, while inflation remains determinate.

4.1.3 The effectiveness and experience with money growth rules

The most effective rule for currency supply chooses: $h_t = p_t^* + \hat{c}_t - \eta(\hat{r}_t + \hat{p}_{t+1}^* - p_t^*) + \hat{u}_t$. This responds to the business cycle and anticipated shifts in the demand and supply for currency. The effectiveness of this policy is given by

$$\varepsilon_t = \frac{1}{1+\eta} \sum_{j=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^j \mathbb{E}_t[\hat{c}_{t+j} - c_{t+j} - \eta(\hat{r}_{t+j} - r_{t+j}) + \eta(p_{t+1+j}^* - \hat{p}_{t+j+1}^*) + (\hat{u}_{t+j} - u_{t+j})]. \tag{36}$$

Just like with interest rate rules, the central bank has to keep track of the state of the economy, but this now involves both the real interest rate and the level of consumption. Even harder, the central bank also needs estimates of u_t, that is, of all changes in the relative usefulness of banknotes relative to their many private substitutes. In advanced economies, financial innovation contributes to large and volatile u_t.

In the early 1980s, the US and the UK both briefly adopted money growth rules. Nominal interest rates were very volatile, as were expected annual inflation rates. An empirical regularity emerged—Goodhart's law—stating that once the central bank started using a policy rule for one measure of h_t, the errors u_t would turn to be even larger than anticipated before. Monetarism is still a useful strategy in scenarios where the central banks lacks credibility in the escape clause of its interest rate rule, so that volatile short-term inflation is tolerable in return for stable long-run inflation.

4.1.4 Nominal rigidities and other breakdowns of the classical dichotomy

With nominal rigidities, the logic behind the determination of the price level with a monetary strategy remains. Now, the changes in money and inflation come with changes in real interest rates and output, as these are all jointly determined. Again, nominal rigidities bring into play an aggregate demand channel, since as households' hold more money, this money chases goods raising aggregate demand, which leads to an increase in production and prices by firms. Sticky prices spread the short-term volatility of inflation that is due to financial shocks into volatility of output.

Monetarism points to alternatives to sticky prices in breaking down the classical dichotomy. If banknotes are used in transactions, their outstanding stock will facilitate trade. In our model, this could be captured by having the utility of money complement the utility from consumption, but there are many better-justified models of these interactions (Lucas and Stokey, 1987, Lagos and Wright, 2005). Changes in the supply of money, by changing the production of goods, would affect both consumption and interest rates in the right-hand side of equation (36). Changes in the supply of banknotes can also interact with financial regulations to affect the creation of bank deposits and bank lending that would affect investment once one allows for the accumulation of capital through financial intermediaries in our model (Brunnermeier and Sannikov, 2016). These additional monetarist channels come with further shocks that could raise inflation volatility.

4.2 Seignorage

Recall that seignorage, as a ratio of consumption, is:

$$\frac{S_t^H}{C_t} = \frac{H_t}{P_t C_t} - \left(\frac{H_{t-1}}{P_{t-1}C_{t-1}}\right)\left(\frac{C_{t-1}}{\Pi_t C_t}\right) \tag{37}$$

Together with equation (31), this expression makes clear that seignorage and inflation are tightly linked. Higher expected inflation comes with higher nominal interest rates, which lowers the demand for currency and lowers seignorage. At the same time, a higher unexpected inflation implies that more goods can be bought with the newly printed banknotes, which raises seignorage.

4.2.1 Seignorage policy rules

The central bank that follows such a rule is committed to generating some revenues, just like a government fiscal agency that has a target for tax revenues, or a State-owned company providing a public service with a target for profits. Historically, this was common, as central banks have been asked for centuries to provide fiscal resources for the sovereign. Only during the past few decades did inflation targeting replace seignorage as the primary task for the central bank.

Given an exogenous target for (log) seignorage s_t^h, the central bank prints more or fewer banknotes as needed to reach this target. Log-linearizing the relation between seignorage and inflation in equation (37) provides a second-order difference equation for the price level. Given an initial p_{-1} and the transversality condition, this equation determines inflation.

4.2.2 Seignorage experience and effectiveness

As a strategy to control inflation, the theory suggest that its effectiveness is poor. In annual data, the large shocks to u_t lead to volatile inflation. In the long run, this approach has often led to hyperinflation. The reason is that in steady state, equation (37) implies that $S \leq C$. If the central bank aims to raise revenue beyond this limit, then inflation is again indeterminate. Furthermore, this upper bound—the peak of the Laffer curve for the inflation tax—is hard to estimate, moves around, and small changes in S close to its peak come with large changes in inflation. Turning the central bank into a fiscal agent often leads to run-away inflation.

This is not a theoretical curiosity, but a common occurrence, especially one that is imposed on the central bank by the Treasury (Sargent and Wallace, 1984). In Latin America in the 1980s and 1990s, the extent to which these fiscal pressures fluctuated from year to year can explain some of the movements in actual inflation rates (Kehoe and Nicolini, 2022). Further back in history, many hyperinflations were associated with seignorage policy rules (Cagan, 1956), and their ends with fiscal reforms that lowered the government's demand for fiscal revenue from the central bank (Sargent, 1982).

4.3 Scarce reserves as money

Most modern central banks stand ready to exchange reserves for banknotes one-to-one at all times, so they only control the sum $V_t + H_t$, the monetary base. People can freely choose to substitute between the two components. So far, we have referred to the interest paid on reserves and the interest on a one-period bond interchangeably, because these two assets were perfect substitutes.

Consider instead a world in which currency provides some service, while reserves do not. As a result, changes in the stock of reserves V_t have no effect on real equilibrium or inflation. Academics refer to this situation as the demand for reserves being satiated, or the market for reserves being saturated. Fed policymakers have come to call it an ample reserves system. Reis (2016) shows evidence for satiation of reserves in the US and argues this is desirable.

Before 2008, this was not the case for the United States, as reserves were scarce and there was a non-horizontal demand for them. In that case, the determination of inflation depends on a hybrid of the monetarist and the no-arbitrage approaches.

4.3.1 Reserves as money

Start with the case where there is a downward-sloping demand for reserves, just as there was one for currency (Diba and Loisel, 2021). Perhaps this will arise because central banks offer digital deposits to households, and not just banks, so that the benefits from using currency for payments will extend to reserves as well.

Let the (log) interest on reserves be i_t^v, while i_t continues to refer to the one-period nominal interest rate. Then, the opportunity cost of holding reserves is the gap $i_t - i_t^v$. We can write a demand curve for reserves as:

$$v_t - p_t = c_t - \eta_v(i_t - i_t^v). \tag{38}$$

and could easily add this to the model in section 1 by introducing a utility from reserves akin to the utility from currency.

Now, the central bank can choose both v_t and i_t^v. In particular, consider the case where it follows a Wicksellian rule, whereby the interest on reserves responds to p_t with a coefficient ϕ. In that case, the price level is determinate as long as $\phi > -1/\eta_v$. This includes the case where $\phi = 0$, that is where there is a pure interest rate peg. The logic is that of the monetarist approach. With two policy tools, the central bank can potentially get closer to tracking the variables it must offset to keep inflation close to its target.

4.3.2 Bank deposits as money

Most households use their bank deposits to engage in transactions. A similar demand equation would hold but with respect to h_t^d, bank deposits, and their opportunity cost is the gap $i_t - i_t^d$ where the interest rate paid on deposits is i_t^d.

The central bank does not control either h_t^d or i_t^d in this case, since both are determined by the equilibrium in the banking sector. However, banks also deposits reserves at the central bank and can invest in financial assets (Piazzesi, Rogers and Schneider, 2022). Optimality in their portfolio choice leads to a log-linearized relation of the form $i_t - i_t^d = \ell(i_t - i_t^v)$. In section 2, competitive frictionless banks in equilibrium implied $\ell = 1$, leading to $i_t = i_t^v$. With market power of banks, or financial frictions, $\ell < 1$. Combining these two equations:

$$h_t^d - p_t = c_t - \eta_d \ell(i_t - i_t^v). \tag{39}$$

If the production of deposits by banks was exogenous with respect to the price level, then by choosing the interest on reserves, the central bank can again control inflation. Even though the policy tool is the interest rate, the economic logic is the monetarist one, as the key equation is the demand curve above, and the terminal condition comes from transversality. If, instead, the quantity of reserves affects the amount of deposits—a money-multiplier process—then we are back at the previous case where the central bank has two tools, v_t and i_t^v, with which to improve the effectiveness of inflation control.

Either way, while central banks' digital currencies, more realistic banking sectors, or scarce reserves all affect the dynamics of inflation, the economic logic and the policy approach by which the central bank can control it are unchanged.

4.4 Pegs

For many emerging and developing economies, the most common approach today to pin down inflation is to peg their currency to another country's currency. This was also the case for most of the advanced world during the Bretton Woods regime, where the peg was to the US dollar between 1944 and 1976 (Bordo, 2017). Until 1971, the US dollar in turn was convertible to gold, following an even older tradition that started in the 1870s of pegging the currency to gold. This is a type of monetarist approach that does not involve banknotes or rules for their supply. Instead, the central bank is following a rule of exchanging domestic reserves for either a commodity or a foreign money.

4.4.1 Commodity pegs

Combining the equality of the marginal rate of substitution across goods in equation (2) with the definition of the price index, we get the log-linearized equation:

$$p_t = \sum_{i=0}^{I} \omega_i p_t(i) = p_t(0) + \sum_{i=1}^{I} \omega_i \rho_t(i). \tag{40}$$

The parameters ω_i, non-negative and summing to one, reflect the weights of each good in the price index, while $\rho(i)$ is the marginal rate of substitution between good i and an arbitrary good 0.

The central bank can announce it will denominate reserves in the units of good 0. Since it can issue reserves in unlimited amounts, the central bank can enforce this denomination by buying good 0 with the reserves and holding it. Seignorage is no longer distributed as dividends to the fiscal authority. This way the central bank can always buy and sell good 0 with reserves to keep their relative price at one forever. This uniquely determines inflation. From equation (40), having defined that $p_t(0) = 1$, the price level p_t is unique. No expectations of the future or terminal conditions are involved, because the central bank is relying on pegging the value of reserves relative to a commodity.

With this strict peg, changes in relative prices would lead the price level to deviate from target. The central bank could adjust the peg to estimates of relative-price movements using a rule $p_t(0) = p_t^* - \sum_{i=1}^{I} \omega_i \hat{\rho}_t(i)$ to improve its effectiveness, which would then be:

$$\varepsilon_t = \sum_{i=1}^{I} \omega_i (\rho_t(i) - \hat{\rho}_t(i)). \tag{41}$$

Changes in the supply of good 0, or in the public's taste for it, become sources of deviations of inflation from target. Moreover, if good 0 is a complement with others in consumption, then the impact on relative prices across all goods can be large. The ideal commodity to peg the price level to has to be storable, have a stable supply, and not be complementary or substitutable with many other goods. Gold or other precious metals meet these criteria and this is why they have often been used with this approach. Still, relative-price movements are large enough that commodity pegs have tended to generate large ε_t (Bordo, 2005).

4.4.2 Exchange rate pegs

Today, it is more common to peg to a foreign currency. This is especially the case in small open economies, which import goods from other countries, often denominated in a dominant foreign currency. A currency board consists of using the same strategy as in a commodity peg, but where reserves are now exchanged for a foreign currency (or a basket of currencies).

The economic logic of how they work is the same. As a simple extension of our model, Assume that aside from $I + 1$ domestic goods, the economy also imports $J + 1$ foreign goods, each with a foreign price $p_t(j)$ (in logs). The exchange rate (also in logs) between the domestic and the foreign units of account is e_t. Letting α denote the measure of home bias, the domestic price level is then equal to:

$$p_t = \alpha \sum_{i=0}^{I} \omega_i p_t(i) + (1-\alpha) \sum_{j=0}^{J} \omega_j (p_t(j) + e_t) = \alpha \sum_{i=0}^{I} \omega_i p_t(i) + (1-\alpha)(p_t^F + e_t) \tag{42}$$

where p_t^F is the price index of the imported goods in foreign currency.

The optimality condition between any two domestic and foreign goods is: $\rho_t(i,j) = p_t(i) - p_t(j) - e_t$, where $\rho_t(i,j)$ is the marginal rate of substitution between consumption of domestic good i and foreign good j. It then follows that: $\sum_{j=0}^{J} \omega_j \rho_t(i,j) = p_t(i) - p_t^F - e_t$. Replacing for $p_t(i)$ in equation (42) delivers:

$$p_t = e_t + p_t^F + \alpha \sum_{i=0}^{I} \sum_{j=0}^{J} \omega_i \omega_j \rho_t(i,j). \tag{43}$$

The second and third term on the right-hand side are exogenous with respect to the price level. An exchange-rate target peg is a choice of e_t. Thus, it uniquely pins down the price level.

The peg of the Hong Kong dollar to the US dollar is perhaps the most famous case of a successful currency peg. In place since October of 1983, the standard deviations of the monthly log nominal exchange rate between the two currencies has been 0.004 over these almost 40 years. And yet, inflation in Hong Kong dollars has been significantly more volatile than inflation in US dollars, peaking at 11% in 1991 and bottoming at -4% in 1999. In part, this happens because choosing a e_t that is constant over time implies that changes in $\rho_t(i,j)$, and consequently in the real exchange rate, lead to wide fluctuations in p_t (Obstfeld and Rogoff, 1995, Ilzetzki, Reinhart and Rogoff, 2019).

Currency boards are rarely adopted for two practical reasons. First, central banks often have conflicting goals for the desired price level π_t^* and for the exchange rate e_t. When pursuing one of them has unpleasant consequences for the other one, the currency board is abandoned.[27] Second, currency boards require that the central bank keeps all the foreign currency it buys with its reserves, so it is ready to buy and sell it as needed. In reality, there is pressure on central banks to distribute some of these assets as dividends, or to exchange the foreign currency for domestic government bonds.

Without foreign assets to back the reserves, countries that try to maintain exchange

[27] See Frankel (2010) for the difficulty of controlling inflation when the central banks cannot commit to a monetary policy strategy.

rate pegs rely either on choosing the interest rate on reserves to mimic movements on foreign interest rates, or in adjusting the supply of money to control its relative scarcity relative to the foreign currency, or a mix of both. This translates into particular interest rate rules or money-growth rules that we have already covered. One way to interpret these pegs is that the value of the exchange rate is seen as a useful indicator of the state of the economy or of the shocks to demand for currency that the central bank aims to track to have a more efficient policy rule. The adoption and abandonment of these pegs follows the usefulness of this indicator as wedges arise between the domestic economy and its foreign counterpart.

5 The solvency approach: dividends and fiscal dominance

The key tool for this approach is the net income, or surplus, of the central bank. The key equation is its intertemporal budget constraint in equation (10).

To start, make the strong assumptions that the net surplus S_t is an exogenous i.i.d. process with mean $\bar{S}$ and that the interest rate on reserves is exogenous $I_t = X^i_t$. Then, equation (10) becomes:

$$P_t = \frac{(1 + X^i_{t-1})(V_{t-1} - P_{t-1}A_{t-1})}{S_t + \bar{S}(\sum_{j=1}^{\infty} \mathbb{E}_t(M_{t,t+j}))}.$$

The sequence of real interest rates and stochastic discount factors is exogenous with respect to the price level. Moreover, V_{t-1} is set in $t-1$, so it is also exogenous with respect to period t realizations. Therefore, the right-hand side is fixed, providing a unique solution for P_t.[28]

The peg for the nominal interest rate X^i_t pins down expected risk-adjusted inflation, as in section 2.1. The budget constraint does the rest, uniquely determining inflation fluctuations. A larger current or future expected surpluses, S_t or $\bar{S}$, lead to a lower price level. By controlling its surpluses, the central bank can target inflation.

The economic force at play is the following: a fall in the surplus of the central bank leaves fewer real resources available to back its debt, reducing its real value. As reserves are default-free, they have a fixed value in nominal terms. Given their role as unit of account, the only way for their real value to fall is for the price level to rise.

[28] The original analysis is Woodford (1994) and Sims (1994) and our approach is closer to that in Cochrane (2005) and Benigno (2020).

The price level adjusts as banks choose to hold more or fewer reserves in response to them becoming a Ponzi scheme. They do so until the real value of reserves is again in line with the central bank's assets and net surplus. It is the control of the real resources earned by the central bank that gives it control over inflation. Insofar as the resources of a government body are fiscal, this mechanism is called the fiscal theory of the price level (FTPL).[29]

5.1 The central bank's net worth and solvency

Assume now that the central bank surplus follows a feedback rule:

$$S_{t+1} = -\phi W_t + X^s_{t+1}. \tag{44}$$

If the net worth falls, the central bank may pay less dividends or cut spending to raise its surplus, in which case $\phi > 0$.

The intertemporal budget constraint of the central bank in equation (10) is equivalent to a no-Ponzi scheme condition: $\lim_{T\to\infty} M_{t,T} W_T \geq 0$ together with a flow budget constraint for the central bank:

$$W_{t+1} = \left[\frac{(1+I_t)P_t}{P_{t+1}}\right] W_t + S_{t+1}\,. \tag{45}$$

Combining these two equations, multiplying by the stochastic discount factor M_{t+1}, and taking expectations as of date t gives a first-order difference equation:

$$\left(1 - \phi(1+R_t)^{-1}\right) W_t = \mathbb{E}_t(M_{t+1}W_{t+1}) - \mathbb{E}_t(M_{t+1}X^s_{t+1}). \tag{46}$$

We can iterate the difference equation forward. If the stochastic discount factor converges to the constant β, then as long as $\phi < \beta^{-1} - 1 < 0$, the terminal condition ensures that W_t will be equal to minus the present value of future exogenous surpluses. But, since $W_t = A_t - V_t/P_t$, if W_t is pinned down, then so is P_t.

Following Leeper (1991), the literature has called feedback rules that satisfy this condition on ϕ *non-Ricardian policies* or active fiscal policies. A central bank that follows these policies will lower its surpluses strongly enough when net worth falls. The price level must rise so that the real value of these reserves falls back into the equilibrium where

[29] For criticism of this mechanism, see Buiter (2017) and for a defense Sims (2013).

the central bank remains solvent. In contrast, Ricardian policies are those for which ϕ is larger than the threshold so the central bank's solvency is assured by it raising its surpluses when net worth falls, no matter what the price level is.

Seemingly paradoxically, inflation control results from the central bank managing its balance sheet in a way that would lead to insolvency by cutting its surpluses just as its net worth falls. Any private agent that tries to do this would find that its liabilities become worthless, so it can get no real resources in exchange for the new debt it issues. What makes the central bank special is that its liabilities are the unit of account. It can honor these liabilities in nominal terms, by just issuing more reserves, but also in real terms as long as the price level adjusts. Therefore, when it follows a rule that would lead to Ponzi scheme for an unchanged price level, the required fall in the real value of reserves requires the price level to rise. Inflation results from the condition that the central bank must stay solvent. It is a capital gain to the central bank that comes at the expense of a capital loss of the private sector holding nominal reserves.

It is easy to derive the effectiveness of this policy as depending on the ability of the central bank to vary the interest rate peg X^i_{t-1} and the net income rule for X^s_{t+j}, given its forecasts of real interest rates and changes in the inflation target. But how can the central bank control its surpluses?

5.2 Dividend rules

Surpluses S_t were derived in section 1 as the sum of four parts. The first is seignorage. A policy rule for seignorage would affect inflation via the supply of currency, as we studied in section 4. Therefore, it could not be used to control it via solvency at the same time.

The second component are the dividends and capital gains from holding assets net of the interest paid on reserves. If the central bank has a "narrow" balance sheet, like the Fed did before 2008, holding almost only short-term bonds, then this term is close to zero. But many central banks hold foreign assets, lend to financial institutions that may not pay back, or hold long-term bonds. Each can generate losses and gains. By choosing the composition of these assets, and so the risk in their returns, the central bank can guide its net surplus and so control inflation. However, since financial returns are volatile, most fluctuations in net surplus and, thus, inflation would be outside the central bank's control. The third component are the expenses of the central banks. These are typically too small to be relevant.

This leaves the fourth component as the main driver: the dividend process $\{D_t\}_{t=0}^{\infty}$ to the Treasury. An independent central bank is one that can choose its dividends. It could use this choice to enforce the surplus rule that leads to inflation control. Yet, many central banks must by law pay out as a dividend all of their net income. While there is great variety in how this net income is calculated, insofar it involves an S_t close to zero, then the determinacy condition is not satisfied (as $\phi = 0$) because the central bank is always solvent. This is sometimes referred to as the central bank having full fiscal support and it rules out the solvency approach to controlling inflation (Hall and Reis, 2015, Del Negro and Sims, 2015, Benigno, 2020).

At another extreme, sometimes the Treasury imposes a dividend process on a no-longer independent central bank. Section 4.2 noted that responding to this imposition by using seignorage pins down the price level through monetarist forces. If instead the central bank responds by issuing reserves to borrow from the private sector and sends the resources to the Treasury, it will be the solvency forces in this section pinning down inflation. Either way, this state of affairs could be referred to as *monetizing the fiscal deficit* since it is the monetary base, currency and reserves, that is adjusting to provide the necessary funding for the Treasury (Reis, 2019*a*).

5.3 Fiscal dominance

The bulk of the literature on the FTPL shifts the focus away from the central bank (Cochrane, 2023, Leeper and Leith, 2016). It starts by noting that the fiscal authorities also face an intertemporal budget constraint linking the value of its liabilities to the present value of its primary surpluses. It then makes three assumptions.

The first is that the government does not default on its liabilities, so that government bonds are perfect substitutes for reserves. Yet, unlike reserves, government bonds are not the unit of account, and sovereign defaults are frequent (Reinhart and Rogoff, 2009). Second, it assumes that dividends are not controlled by the central bank and can take any value. In that case, the intertemporal budgets of the central bank and the Treasury are not two separate constraints, but rather a single consolidated constraint that states that the sum of reserves and government liabilities (netting out the government bonds held by the central bank) equals the present value of surpluses of both the government and the central bank (but netting out the central bank's dividends). The third assumption is that the Treasury solely chooses this surplus, and any actions of the central bank on its expenses, seignorage or composition of its assets is precisely offset by the Treasury.

Combining the three assumptions, the same logic that allowed the central bank to control inflation, is now applied to the Treasury. That is, the solvency of the Treasury becomes tied to the price level. Because the control errors arise from the side of the government and its fiscal surplus, the effectiveness of this approach to pin down inflation will be very poor. In the United States, even as other policy strategies were followed over the decades, changes in the market value of public debt in the background were consistent with the ups and downs of inflation (Cochrane, 2022*a*), more noticeable during the high inflation of the 1970s that coincided with large and persistent fiscal deficits (Sims, 2011, Bianchi and Melosi, 2018).

5.4 Nominal rigidities

Sticky prices and nominal rigidities do not alter the underlying logic of the solvency approach. In terms of the basic new Keynesian model, the equilibrium condition that is combined with the IS and Phillips curve in equations (19)-(20), is a linearized version of the central bank flow of funds in equation (10) stating that net worth increases with surpluses and falls with the real interest rate. This provides a system of three equations with three unknowns—the output gap, inflation, and net worth—as a function of nominal interest rates and surpluses. Instead of a feedback interest rate rule, as in section 2, or a money growth rule, like in section 4, now there will be two parts of the policy rule: an exogenous process for the nominal interest rate peg set by the central bank, and a rule for the central bank's surpluses as in equation (44). The first part fixes expected inflation, while the second part determines its response to shocks.

As before, nominal rigidities add complementary channels working through output and, especially, real interest rates. Now, after a loss, the jump in inflation to reestablish the solvency of the central bank will lower the real interest rate with a nominal interest rate peg. This stimulates more consumption, which further raises inflation via aggregate demand. In turn, the transfer of wealth for the private sector towards the central bank happens not just via inflation but also because of the lower real interest rates over time. Therefore, the dynamics of inflation change relative to when the classical dichotomy holds, becoming more drawn out.

These dynamics feed back into the size of the inflation response because the real value of the central bank's net worth depends on the maturity of its assets. The value of long-dated assets depends on both unexpected inflation and on the path of real interest rates. Quantitative easing strategies that set the maturity of the central bank's assets are a key determinant of the persistence of inflation deviations from target (Cochrane, 2001).

5.5 Alternative mechanisms to break the classical dichotomy

The focus on solvency brings to light different mechanisms that can break the classical dichotomy even with flexible prices. The transfer of wealth to the central bank comes at the expense of the banks holding these reserves (Reis, 2016). If banks' net worth constrains their willingness to extend private credit, then this provides a credit channel that complements the ones we already discussed in section 3.

More generally, in an economy where different agents hold different mixes of nominal and real assets, the inflation that is driven by the central bank's solvency will induce redistributions of wealth. With incomplete markets, these will matter for aggregate demand and so for the dynamics of inflation (Auclert, 2019, Kaplan, Nikolakoudis and Violante, 2023).

Incomplete markets raise another channel for breaking the classical dichotomy through the solvency channel. The total stock of reserves outstanding is a relevant variable for inflation because it determines by how much inflation must change to keep the real value of these reserves in line with solvency. Under the fiscal dominance version, the size of the government debt matters as well. In incomplete markets models, the net supply of assets available to agents affects their equilibrium choices. The reason is that, unable to diversify individual risks, agents engage in precautionary savings through these assets, so their relative availability determines the cost and limits of doing so. When inflation results from the central bank's choices regarding its surplus, this both ex post redistributes wealth across different agents and ex ante affects the expected returns on different savings vehicles. Both affect the desire to consume and produce i.e. real outcomes in the economy (Hagedorn, 2018*a*,*b*).

In the other direction, breaking the classical dichotomy affects the source of shocks to inflation. Changes in output and inflation directly affect the demand for currency and the seignorage revenue of the government. Changes in the path of real interest rates over time affect the capital gains and losses in the central bank's portfolio. And, the effect of monetary policy on real outcomes has an effect on tax collections and government spending and so on the primary surpluses of the fiscal authority. Through fiscal dominance, these would trigger changes in the demand for dividends from the central bank (Reis, 2019*a*).

6 Conclusion: a unified approach and the 2021-... inflation disaster

Each of the previous sections emphasized one policy tool that leans on one particular economic force to bring inflation close to target. They all co-existed in the same dynamic general equilibrium, not as contradictory theories, but as different policy options. The central bank can choose a strategy that relies on arbitrage forces, on money market forces, or on solvency forces, and within each it can choose one rule to implement it, whether it is an interest rate rule, or a money growth rule, or a net surplus rule, or one of their many variants.

Importantly, the central bank can choose only *one* of them. Otherwise, the different policies are in conflict with each other; mathematically the economic system is over-determined. The literature has called the policy strategy that is followed the *active* one. The others are *passive*. So, if the central bank chooses the no-arbitrage approach setting interest rates to control inflation, then that is active, while the amount of currency it prints or the net surpluses it generates are all passive.[30]

We derived measures of effectiveness for each policy strategy. Our hope is that the academic debate shifts from arguments on which assumptions are perceived as being more convincing to attempts to measure the maximum effectiveness of each strategy. We also provided our own take of the history of monetary policy and inflation in advanced economies in the post war.

During Bretton Woods, most countries followed a monetarist strategy by pegging to the US dollar. During the 1970s, the US or the UK followed an interest rate strategy without satisfying the conditions for determinacy, both in the responsiveness of the feedback rules and in the role of the escape clause. This led in the early 1980s to both countries having brief experiences with money growth rules. At the same time, in countries through Latin America, a combination of monetarist seignorage rules or solvency strategies imposed by fiscal authorities led to high and volatile inflation.

The conquest of stable inflation between 1990 and 2010 came from a coherent strategy across the different elements. Central banks used feedback interest rate rules that satisfied the determinacy principle. They adopted inflation targeting regimes to manage

[30] A different use of the word active and passive is to describe which of the two institutions, the central bank or the Treasury, is imposing its decisions on the other. If they are playing a game with each other, this will affect how the policy approach is chosen and set. Unfortunately, both definitions of active/passive are used in the literature, generating confusion.

expectations, while having escape clauses reliant on monetary anchors. Central bank independence imposed rules on the dividends paid to Treasuries that kept the central bank solvent at all times. These ruled out seignorage or insolvency from driving inflation. Between 2010 and 2020 this framework was refined through forward guidance, going long, and communication strategies to overcome the effective lower bound, even if these tools relied heavily on rationality of expectations and were generally less effective.

Between 2021 and 2023, this conquest was lost. While the unusual shocks that hit the economy justified an optimal inflation rate well above 2%, the actual inflation rate was much higher than that. Why this happened is understandably still the subject of debate. Perhaps central banks kept to this same framework but they made mistakes in judging the state of the economy, in anchoring expectations, and in using unconventional interest rate tools that are less effective (Reis, 2023, Eggertsson and Kohn, 2023). Or, perhaps the expansion of the balance sheet of central banks through quantitative easing and the large increase in public debt during the pandemic have made central bank independence untenable and it is concerns about solvency that are driving inflation (Bianchi and Melosi, 2022, Cochrane, 2022*b*). The future will show.

References

Adão, Bernardino, Isabel Correia, and Pedro Teles. 2014. "Short and Long Interest Rate Targets." *Journal of Monetary Economics*, 66: 95–107.

Agarwal, Ruchir, and Miles Kimball. 2019. "Enabling Deep Negative Rates to Fight Recessions: A Guide." IMF Working Paper 19/84.

Alvarez, Fernando, and Francesco Lippi. 2014. "Persistent Liquidity Effects and Long-Run Money Demand." *American Economic Journal: Macroeconomics*, 6(2): 71–107.

Andrade, Philippe, and Filippo Ferroni. 2021. "Delphic and Odyssean Monetary Policy Shocks: Evidence from the Euro Area." *Journal of Monetary Economics*, 117: 816–832.

Angeletos, George-Marios, and Chen Lian. 2018. "Forward Guidance without Common Knowledge." *American Economic Review*, 108(9): 2477–2512.

Angeletos, George-Marios, and Chen Lian. 2023. "Determinacy without the Taylor Principle." *Journal of Political Economy*, 131(8): 2125 – 2164.

Aruoba, Boragan, Pablo Cuba-Borda, and Frank Schorfheide. 2017. "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries." *Review of Economic Studies*, 85(1): 87–118.

Atkeson, Andrew, Varadarajan V Chari, and Patrick J Kehoe. 2010. "Sophisticated Monetary Policies." *Quarterly Journal of Economics*, 125(1): 47–89.

Auclert, Adrien. 2019. "Monetary Policy and the Redistribution Channel." *American Economic Review*, 109(6): 2333–2367.

Ball, Laurence. 2001. "Another Look at Long-Run Money Demand." *Journal of Monetary Economics*, 47(1): 31–44.

Bassetto, Marco. 2005. "Equilibrium and Government Commitment." *Journal of Economic Theory*, 124(1): 79–105.

Benassy, Jean-Pascal. 2000. "Price Level Determinacy under a Pure Interest Rate Peg." *Review of Economic Dynamics*, 3(1): 194–211.

Benati, Luca, Robert E. Lucas Jr., Juan Pablo Nicolini, and Warren E. Weber. 2021. "International Evidence on Long-Run Money Demand." *Journal of Monetary Economics*, 117: 43–63.

Benhabib, Jess, Stephanie Schmitt-Grohe, and Martin Uribe. 2001. "The Perils of Taylor Rules." *Journal of Economic Theory*, 96(1-2): 40–69.

Benigno, Pierpaolo. 2020. "A Central Bank Theory of Price Level Determination." *American Economic Journal: Macroeconomics*, 12(3): 258–283.

Berger, David, Konstantin Milbradt, Fabrice Tourre, and Joseph Vavra. 2021. "Mortgage Prepayment and Path-Dependent Effects of Monetary Policy." *American Economic Review*, 111(9): 2829–2878.

Bernanke, Ben S. 2020. "The New Tools of Monetary Policy." *American Economic Review*, 110(4): 943–983.

Bernanke, Ben S, and Frederic S Mishkin. 1997. "Inflation Targeting: A New Framework for Monetary Policy?" *Journal of Economic Perspectives*, 11(2): 97–116.

Bernanke, Ben S, Mark Gertler, and Simon Gilchrist. 1999. "Issues in the Design of Monetary Policy Rules." In *Handbook of Macroeconomics*. Vol. 1, , ed. J. B. Taylor and M. Woodford, 1341–1393.

Bianchi, Francesco, and Leonardo Melosi. 2018. "Constrained Discretion and Central Bank Transparency." *Review of Economics and Statistics*, 100(1): 187–202.

Bianchi, Francesco, and Leonardo Melosi. 2022. "Inflation as a Fiscal Limit." FRB Chicago Working Paper 37.

Bilbiie, Florin. 2018. "Monetary Policy and Heterogeneity: An Analytical Framework." CEPR Discussion Paper 12601.

Blinder, Alan S., and Ricardo Reis. 2005. "Understanding the Greenspan standard." In *The Greenspan Era: Lessons for the Future*. 11–96. Jackson Hole Symposium:Federal Reserve Bank of Kansas City.

Blinder, Alan S, Michael Ehrmann, Marcel Fratzscher, Jakob De Haan, and David-Jan Jansen. 2008. "Central Bank Communication and Monetary Policy: A Survey of Theory and Evidence." *Journal of Economic Literature*, 46(4): 910–945.

Boneva, Lena Mareen, R. Anton Braun, and Yuichiro Waki. 2016. "Some Unpleasant Properties of Loglinearized Solutions when the Nominal Rate Is Zero." *Journal of Monetary Economics*, 84: 216–232.

Bordo, Michael. 2005. *The Gold Standard and Related Regimes: Collected Essays.* Cambridge University Press.

Bordo, Michael D. 2017. "The Operation and Demise of the Bretton Woods System; 1958 to 1971." NBER Working Paper 23189.

Brunnermeier, Markus K, and Yuliy Sannikov. 2016. "The I Theory of Money." NBER Working Paper 22533.

Buiter, Willem H. 2017. "The Good and the Bad Fiscal Theory of the Price Level." CEPR Discussion Paper 11975.

Bullard, James, and Kaushik Mitra. 2002. "Learning about Monetary Policy Rules." *Journal of Monetary Economics*, 49(6): 1105–1129.

Cagan, Phillip. 1956. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory if Money.* , ed. Milton Friedman, 25–117.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12(3): 383–398.

Campbell, Jeffrey R, Jonas DM Fisher, Alejandro Justiniano, and Leonardo Melosi. 2017. "Forward Guidance and Macroeconomic Outcomes since the Financial Crisis." *NBER Macroeconomics Annual*, 31(1): 283–357.

Canzoneri, Matthew B., Dale W. Henderson, and Kenneth S. Rogoff. 1983. "The Information Content of the Interest Rate and Optimal Monetary Policy." *Quarterly Journal of Economics*, 98(4): 545–566.

Capie, Forrest. 2010. *The Bank of England: 1950s to 1979.* Cambridge University Press.

Carlstrom, Charles T, and Timothy S Fuerst. 2002. "Taylor Rules in a Model that Satisfies the Natural-Rate Hypothesis." *American Economic Review*, 92(2): 79–84.

Carlstrom, Charles T, and Timothy S Fuerst. 2003. "Money Growth Rules and Price Level Determinacy." *Review of Economic Dynamics*, 6(2): 263 – 275.

Carlstrom, Charles T., Timothy Fuerst, and Matthias Paustian. 2015. "Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg." *Journal of Monetary Economics*, 76: 230–243.

Carvalho, Carlos, Fernanda Nechio, and Tiago Tristao. 2021. "Taylor Rule Estimation by OLS." *Journal of Monetary Economics*, 124: 140–154.

Cecchetti, Stephen G, Peter Hooper, Bruce C Kasman, Kermit L Schoenholtz, and Mark W Watson. 2007. "Understanding the Evolving Inflation Process." US Monetary Policy Forum.

Christiano, Lawrence J., and Massimo Rostagno. 2001. "Money Growth Monitoring and the Taylor Rule." NBER Working Paper 8539.

Christiano, Lawrence J., and Yuta Takahashi. 2018. "Discouraging Deviant Behavior in Monetary Economics." NBER Working Paper 24949.

Christiano, Lawrence J., Martin S Eichenbaum, and Benjamin K Johannsen. 2018. "Does the New Keynesian Model Have a Uniqueness Problem?" NBER Working Paper 24612.

Christiano, Lawrence J, Mathias Trabandt, and Karl Walentin. 2010. "DSGE Models for Monetary Policy Analysis." In *Handbook of Monetary Economics*. Vol. 3, 285–367.

Ciccarelli, Matteo, Angela Maddaloni, and José-Luis Peydró. 2015. "Trusting the-Bankers: A new Look at the Credit Channel of Monetary Policy." *Review of Economic Dynamics*, 18(4): 979–1002.

Clarida, Richard H, and Mark Gertler. 1997. "How the Bundesbank Conducts Monetary Policy." In *Reducing Inflation: Motivation and Strategy*. , ed. Christina D Romer and David H Romer, 363–412.

Clarida, Richard, Jordi Galí, and Mark Gertler. 2000*a*. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics*, 115(1): 147–180.

Clarida, Richard, Jordi Gali, and Mark Gertler. 2000*b*. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *The Quarterly Journal of Economics*, 115(1): 147–180.

Cochrane, John H. 2001. "Long-term Debt and Optimal Policy in the Fiscal Theory of the Price Level." *Econometrica*, 69(1): 69–116.

Cochrane, John H. 2005. "Money as Stock." *Journal of Monetary Economics*, 52(3): 501–528.

Cochrane, John H. 2011. "Determinacy and Identification with Taylor Rules." *Journal of Political Economy*, 119(3): 565–615.

Cochrane, John H. 2017. "The New-Keynesian Liquidity Trap." *Journal of Monetary Economics*, 92: 47–63.

Cochrane, John H. 2022*a*. "Fiscal Histories." *Journal of Economic Perspectives*, 36(4): 125–146.

Cochrane, John H. 2022*b*. "Inflation Past, Present and Future: Fiscal Shocks, Fed Response, and Fiscal Limits." NBER Working Paper 30096.

Cochrane, John H. 2023. *The Fiscal Theory of the Price Level.* Princeton University Press.

Coibion, Olivier, and Yuriy Gorodnichenko. 2011. "Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation." *American Economic Review*, 101(1): 341–370.

Del Negro, Marco, and Christopher A. Sims. 2015. "When Does a Central Bank's Balance Sheet Require Fiscal Support?" *Journal of Monetary Economics*, 73: 1–19.

Del Negro, Marco, Marc P Giannoni, and Christina Patterson. 2012. "The Forward Guidance Puzzle." FRB of New York Staff Report 574.

Den Haan, Wouter J, Pontus Rendahl, and Markus Riegler. 2017. "Unemployment (Fears) and Deflationary Spirals." *Journal of the European Economic Association*, 16(5): 1281–1349.

Diba, Behzad, and Olivier Loisel. 2021. "Pegging the Interest Rate on Bank Reserves: A Resolution of New Keynesian Puzzles and Paradoxes." *Journal of Monetary Economics*, 118: 230–244.

Dincer, Nergiz, Barry Eichengreen, Petra Geraats, et al. 2022. "Trends in Monetary Policy Transparency: Further Updates." *International Journal of Central Banking*, 18(1): 331–348.

Eggertson, Gauti B., and Vaishali Garga. 2019. "Sticky Prices versus Sticky Information: Does it Matter for Policy Paradoxes?" *Review of Economic Dynamics*, 31: 363–392.

Eggertsson, Gauti B, and Donald Kohn. 2023. "The Inflation Surge of the 2020s: The Role of Monetary Policy." Hutchins Center Working Paper 87.

Eggertsson, Gauti B., and Michael Woodford. 2003. "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity*, 34(1): 139–235.

Evans, George W., and Garey Ramey. 1992. "Expectation Calculation and Macroeconomic Dynamics." *American Economic Review*, 82(1): 207–224.

Evans, George W., and Seppo Honkapohja. 2001. *Learning and Expectations in Macroeconomics.* Princeton University Press.

Fabo, Brian, Martina Jancokova, Elisabeth Kempf, and Lubos Pastor. 2021. "Fifty Shades of QE: Comparing Findings of Central Bankers and Academics." *Journal of Monetary Economics*, 120: 1–20.

Farhi, Emmanuel, and Iván Werning. 2019. "Monetary Policy, Bounded Rationality, and Incomplete Markets." *American Economic Review*, 109(11): 3887–3928.

Fernández-Villaverde, Jesús, Grey Gordon, Pablo Guerrón-Quintana, and Juan F. Rubio-Ramírez. 2015. "Nonlinear Adventures at the Zero Lower Bound." *Journal of Economic Dynamics and Control*, 57: 182 – 204.

Fiore, Fiorella De, and Oreste Tristani. 2013. "Optimal Monetary Policy in a Model of the Credit Channel." *The Economic Journal*, 123(571): 906–931.

Frankel, Jeffrey. 2010. "Monetary Policy in Emerging Markets: A Survey." In *Handbook of Monetary Economics*. Vol. 3, , ed. Benjamin M. Friedman and Michael Woodford, 1439–1520.

Gabaix, Xavier. 2019. "A Behavioral New Keynesian Model." NBER Working Paper 22954.

Gabaix, Xavier. 2020. "A Behavioral New Keynesian Model." *American Economic Review*, 110(8): 2271–2327.

García-Schmidt, Mariana, and Michael Woodford. 2019. "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis." *American Economic Review*, 109(1): 86–120.

Gertler, Mark, and Peter Karadi. 2015. "Monetary Policy Surprises, Credit Costs, and Economic Activity." *American Economic Journal: Macroeconomics*, 7(1): 44–76.

Goodfriend, Marvin. 2016. "The Case for Unencumbering Interest Rate Policy at the Zero Bound." In *Designing Resilient Monetary Policy Frameworks for the Future*. 127–160. Jackson Hole Symposium:Federal Reserve Bank of Kansas City.

Greenwald, Daniel. 2018. "The Mortgage Credit Channel of Macroeconomic Transmission." Available at: https://www.dlgreenwald.com/upload\s/4/5/2/8/45280895/mortgage_credit_\channel_feb_2018.pdf.

Gürkaynak, Refet, Brian Sack, and Eric Swanson. 2005. "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements." *International Journal of Central Banking*, 1(1).

Hagedorn, Marcus. 2018*a*. "A Demand Theory of the Price Level." CEPR Discussion Paper 11364.

Hagedorn, Marcus. 2018*b*. "Prices and Inflation when Government Bonds are Net Wealth." CEPR Discussion Paper 12769.

Hall, Robert E. 1997. "Irving Fisher's Self-Stabilizing Money." *American Economic Review Papers and Proceedings*, 87(2): 436–438.

Hall, Robert E., and Ricardo Reis. 2015. "Maintaining Central-Bank Solvency under New-Style Central Banking." NBER Working Paper 21173.

Hall, Robert E., and Ricardo Reis. 2016. "Achieving Price Stability by Manipulating the Central Bank's Payment on Reserves." NBER Working Paper 22761.

Hartmann, Philipp, and Frank Smets. 2018. "The European Central Bank's Monetary Policy during its First 20 Years." *Brookings Papers on Economic Activity*, 2018(2): 1–146.

Hume, David. 1752. *Political Discourses.* A. Kincaid & A. Donaldson.

Ilzetzki, Ethan, Carmen M. Reinhart, and Kenneth S. Rogoff. 2019. "Exchange Arrangements Entering the Twenty-First Century: Which Anchor Will Hold?" *The Quarterly Journal of Economics*, 134(2): 599–646.

IMF, I. 2020. "The Central Bank Transparency Code." IMF Policy Paper 20/38.

Ireland, Peter N. 2009. "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand." *American Economic Review*, 99(3): 1040–1052.

Kaplan, Greg, Georgios Nikolakoudis, and Giovanni L Violante. 2023. "Price Level and Inflation Dynamics in Heterogeneous Agent Economies." NBER Working Paper 31433.

Kehoe, Timothy J, and Juan Pablo Nicolini. 2022. *A Monetary and Fiscal History of Latin America, 1960–2017.* U of Minnesota Press.

Khan, Aubhik, Robert G. King, and Alexander L. Wolman. 2003. "Optimal Monetary Policy." *Review of Economic Studies*, 70(4): 825–860.

Kiley, Michael. 2016. "Policy Paradoxes in the New-Keynesian Model." *Review of Economic Dynamics*, 21: 1–15.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2013. "The Ins and Outs of LSAPs." 57–111. Jackson Hole Symposium:Federal Reserve Bank of Kansas City.

Lagos, Ricardo, and Randall Wright. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, 113(3): 463–484.

Leeper, Eric M. 1991. "Equilibria under Active and Passive Monetary and Fiscal Policies." *Journal of Monetary Economics*, 27: 129–147.

Leeper, Eric M, and Campbell Leith. 2016. "Understanding Inflation as a Joint Monetary–Fiscal Phenomenon." In *Handbook of Macroeconomics*. Vol. 2, , ed. John B. Taylor and Harald Uhlig, 2305–2415.

Leeson, Robert, Evan F Koenig, and George A Kahn. 2013. *The Taylor Rule and the Transformation of Monetary policy.* Hoover Press.

Loisel, Olivier. 2021. "The Implementability and Implementation of Feasible Paths by Stabilization Policy." *Theoretical Economics*, 16(2): 677–716.

Lubik, Thomas A., and Frank Schorfheide. 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review*, 94(1): 190–217.

Lucas, Robert, and Nancy Stokey. 1987. "Money and Interest in a Cash-in-Advance Economy." *Econometrica*, 55(3): 491–513.

Magill, Michael, and Martine Quinzii. 2014. "Term Structure and Forward Guidance as Instruments of Monetary Policy." *Economic Theory*, 56(1): 1–32.

McCallum, Bennett T. 1981. "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations." *Journal of Monetary Economics*, 8(3): 319–329.

McCallum, Bennett T. 1999. "Issues in the Design of Monetary Policy Rules." In *Handbook of Macroeconomics*. Vol. 1, , ed. J. B. Taylor and M. Woodford, Chapter 23, 1483–1530.

McCallum, Bennett T. 2003. "Multiple-Solution Indeterminacies in Monetary Policy Analysis." *Journal of Monetary Economics*, 50(5): 1153–1175.

McGough, Bruce, Glenn D. Rudebusch, and John C. Williams. 2005. "Using a Long-Term Interest Rate as the Monetary Policy Instrument." *Journal of Monetary Economics*, 52(5): 855 – 879.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson. 2016. "The Power of Forward Guidance Revisited." *American Economic Review*, 106(10): 3133–58.

Meltzer, Allan H. 2010. *A History of the Federal Reserve, Volume 2: 1970–1986.* University of Chicago Press.

Mertens, Karel R. S. M., and Morten O. Ravn. 2014. "Fiscal Policy in an Expectations-Driven Liquidity Trap." *Review of Economic Studies*, 81(4): 1637–1667.

Meyer-Gohde, Alexander, and Mary Tzaawa-Krenzler. 2023. "Sticky Information and the Taylor Principle." Unpublished manuscript.

Muth, John. 1960. "Optimal Properties of Exponentially Weighted Forecasts." *Journal of the American Statistical Association*, 55: 290–306.

Nakajima, Tomoyuki, and Herakles Polemarchakis. 2005. "Money and Prices under Uncertainty." *Review of Economic Studies*, 72(1): 223–246.

Obstfeld, Maurice, and Kenneth Rogoff. 1983. "Speculative Hyperinflations in Maximizing Models: Can we Rule them Out?" *Journal of Political Economy*, 91(4): 675–687.

Obstfeld, Maurice, and Kenneth Rogoff. 1995. "The Mirage of Fixed Exchange Rates." *The Journal of Economic Perspectives*, 9(4): 73–96.

Orphanides, Athanasios. 2003. "The Quest for Prosperity Without Inflation." *Journal of Monetary Economics*, 50(3): 633–663.

Orphanides, Athanasios. 2019. "Monetary Policy Strategy and its Communication." 22–24. Jackson Hole Symposium:Federal Reserve Bank of Kansas City.

Piazzesi, Monika, Ciaran Rogers, and Martin Schneider. 2022. "Money and Banking in a New Keynesian Model." Available at: https://web.stanford.edu/~piazzesi/MB\inNK.pdf.

Reinhart, Carmen M., and Kenneth S. Rogoff. 2009. *This Time is Different: Eight Centuries of Financial Folly.* Princeton University Press.

Reis, Ricardo. 2013. "Central Bank Design." *Journal of Economic Perspectives*, 27(4): 17–44.

Reis, Ricardo. 2016. “Funding Quantitative Easing to Target Inflation.” In *Designing Resilient Monetary Policy Frameworks for the Future*. Jackson Hole Symposium:Federal Reserve Bank of Kansas City.

Reis, Ricardo. 2019*a*. “Can the Central Bank Alleviate Fiscal Burdens?” In *The Economics of Central Banking*. *Oxford University Press Handbooks in Economics*, , ed. David Mayes, Peter Siklos and Jan-Egbert Strum. Oxford University Press.

Reis, Ricardo. 2019*b*. “Central Banks Going Long.” In *Monetary Policy and Financial Stability: Transmission Mechanisms and Policy Implications*. , ed. Markus Brunnermeier Alvaro Aguirre and Diego Saravia, 43–81. Central Bank of Chile.

Reis, Ricardo. 2021. “Losing the Inflation Anchor.” *Brookings Papers on Economic Activity*, 307–361.

Reis, Ricardo. 2022. “Expected Inflation in the Euro Area: Measurement and Policy Responses.” In *ECB Forum on Central Banking*.

Reis, Ricardo. 2023. “The Burst of High Inflation in 2021-22: How and Why Did We Get Here?” In *How Monetary Policy Got Behind the Curve–And How to Get it Back*. , ed. Michael Bordo, John Cochrane and John Taylor, 203–252. Hoover Institution Press.

Rogoff, Kenneth. 2017. “Dealing with Monetary Paralysis at the Zero Bound.” *Journal of Economic Perspectives*, 31(3): 47–66.

Rostagno, Massimo, Carlo Altavilla, Giacomo Carboni, Wolfgang Lemke, Roberto Motto, Arthur Saint Guilhem, and Jonathan Yiangou. 2021. *Monetary Policy in Times of Crisis: A Tale of Two Decades of the European Central Bank.* Oxford University Press.

Rotemberg, Julio. 1982. “Monopolistic Price Adjustment and Aggregate Output.” *Review of Economic Studies*, 44(4): 517–531.

Sargent, Thomas J. 1982. “The Ends of Four Big Inflations.” In *Inflation: Causes and Effects*. 41–98. National Bureau of Economic Research.

Sargent, Thomas J, and Neil Wallace. 1975. “”Rational” Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule.” *Journal of Political Economy*, 83(2): 241–254.

Sargent, Thomas J, and Neil Wallace. 1984. "Some Unpleasant Monetarist Arithmetic." In *Monetarism in the United Kingdom*. 15–41. Springer.

Sims, Christopher A. 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory*, 4(3): 381–399.

Sims, Christopher A. 2011. "Stepping on a rake: the role of fiscal policy in the inflation of the 1970s." *European Economic Review*, 55: 48–56.

Sims, Christopher A. 2013. "Paper Money." *American Economic Review Papers and Proceedings*, 103(3): 563–584.

Taylor, Jhn B., ed. 1999. *Monetary Policy Rules.* University of Chicago Press.

Taylor, John. 1996. "Policy Rules as a Means to a More Effective Monetary Policy." *Monetary and Economic Studies*, 14(1): 28–39.

Taylor, John B. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, 39: 195–214.

Werning, Ivan. 2011. "Managing a Liquidity Trap: Monetary and Fiscal Policy." NBER Working Paper 17344.

Woodford, Michael. 1994. "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy." *Economic Theory*, 4(3): 345–380.

Woodford, Michael. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Woodford, Michael. 2010. "Optimal Monetary Stabilization Policy." In *Handbook of Monetary Economics*. Vol. 3, , ed. Benjamin M. Friedman and Michael Woodford, Chapter 14, 723 – 828. Elsevier.

Woodford, Michael. 2013. "Macroeconomic Analysis Without the Rational Expectations Hypothesis." *Annual Review of Economics*, 5: 303–346.